Merging-Style Problems

Pei-yih Ting

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- LC313 Super Ugly Number
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  逆序數 (逆序對backward sequence個數)
- LC327 Count of Range Sum
- 其他應用

合併排序法 (Merge Sort)

- 以一個範例來說明基本的演算法

9 3 4 220 1 3 10 5 8 7 2

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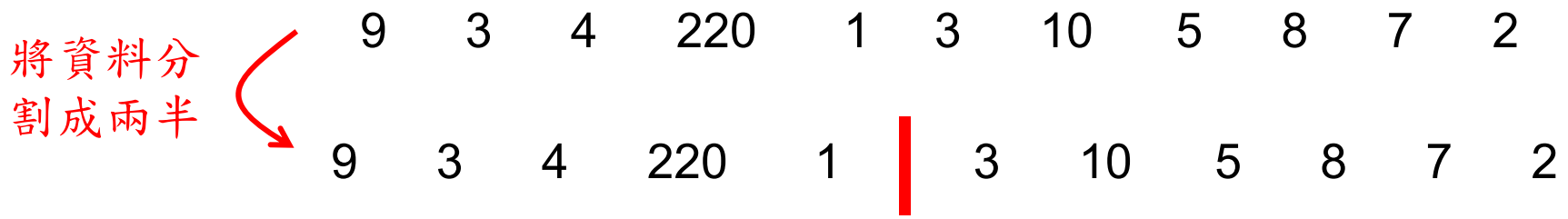

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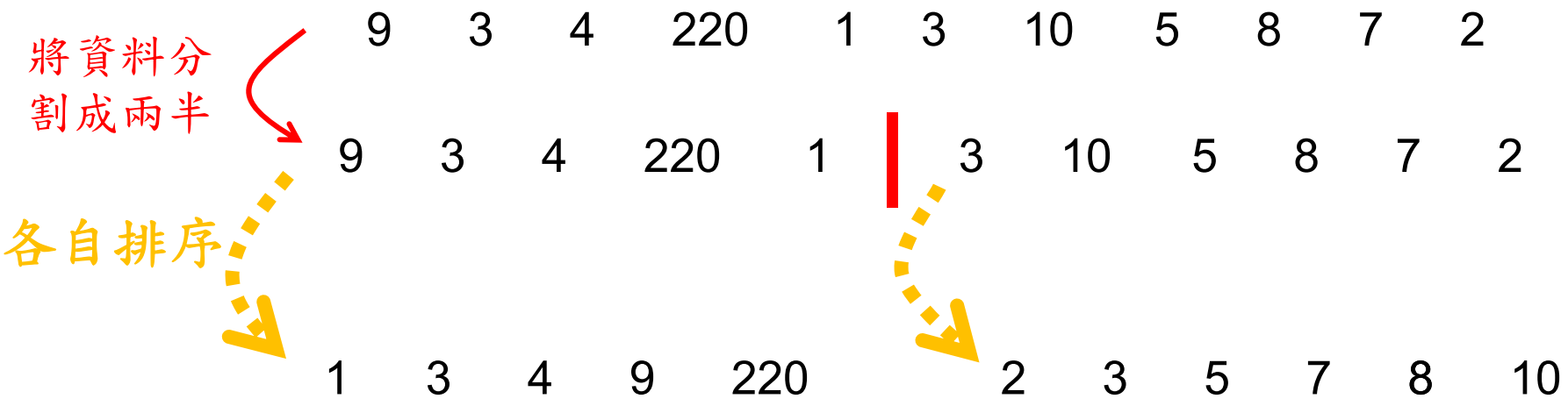

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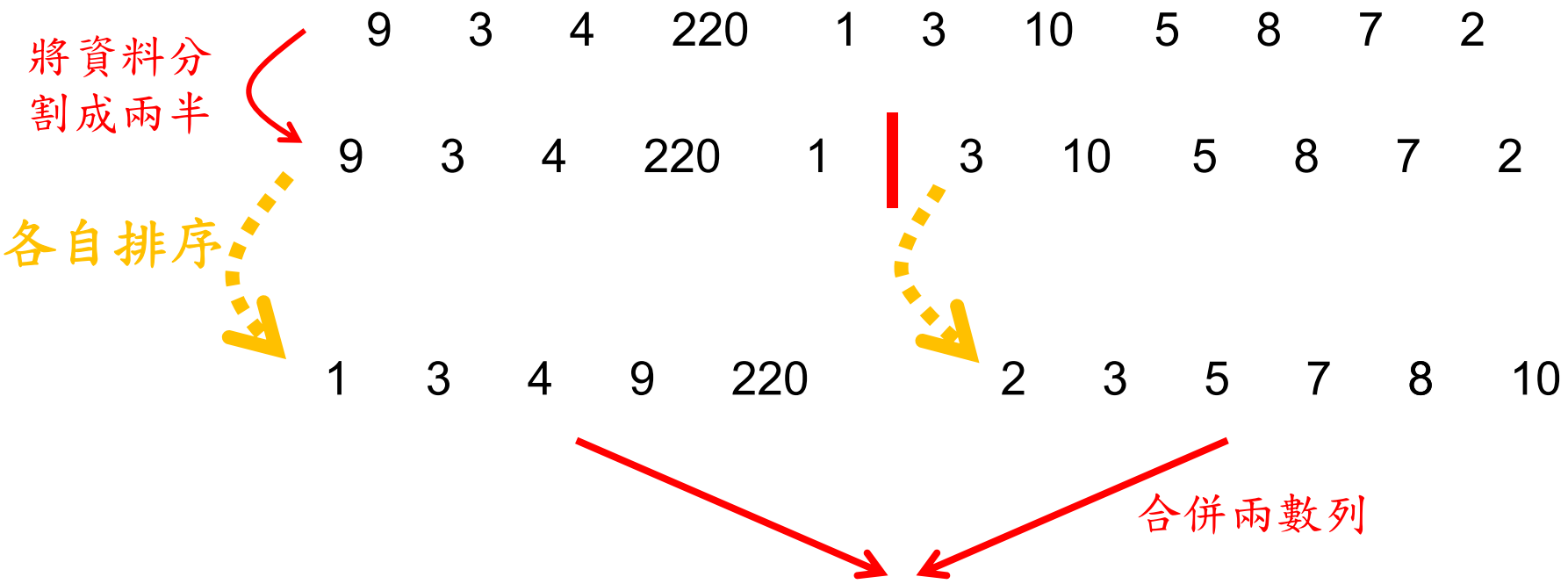

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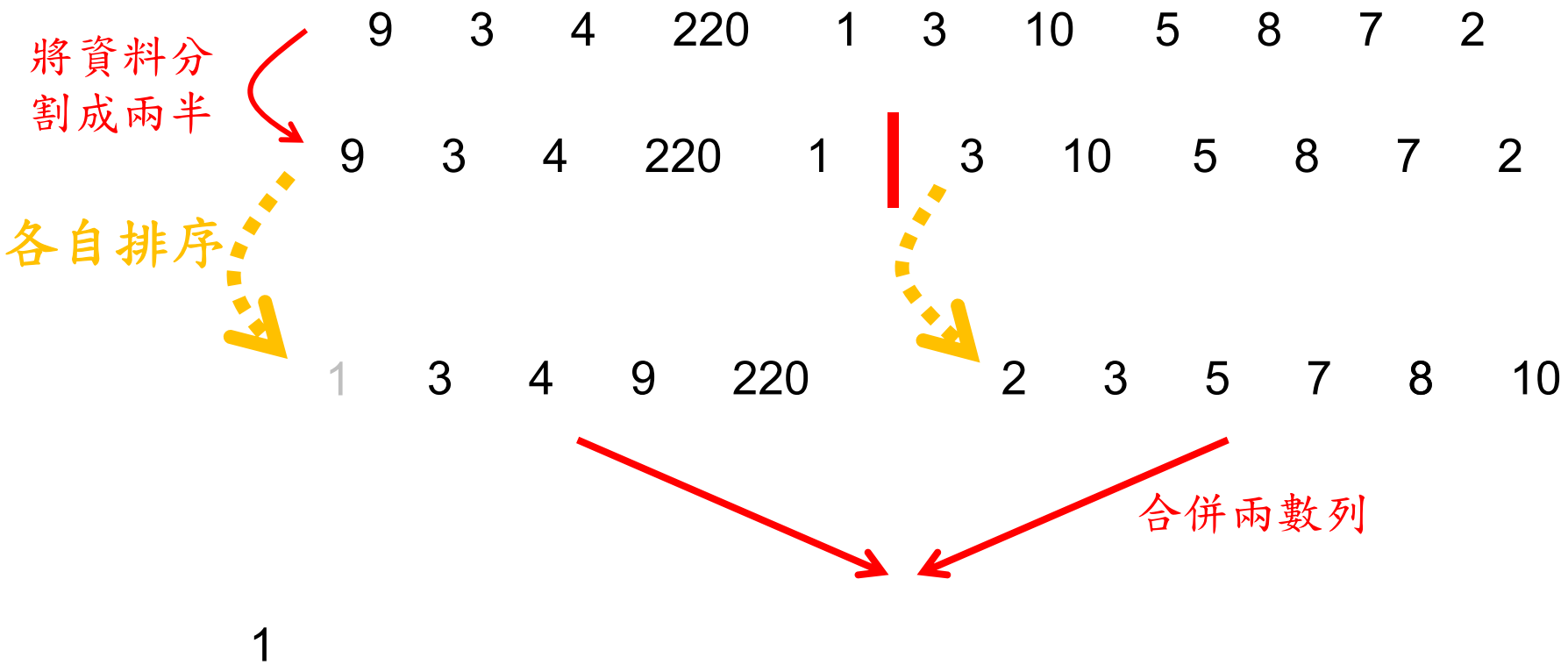

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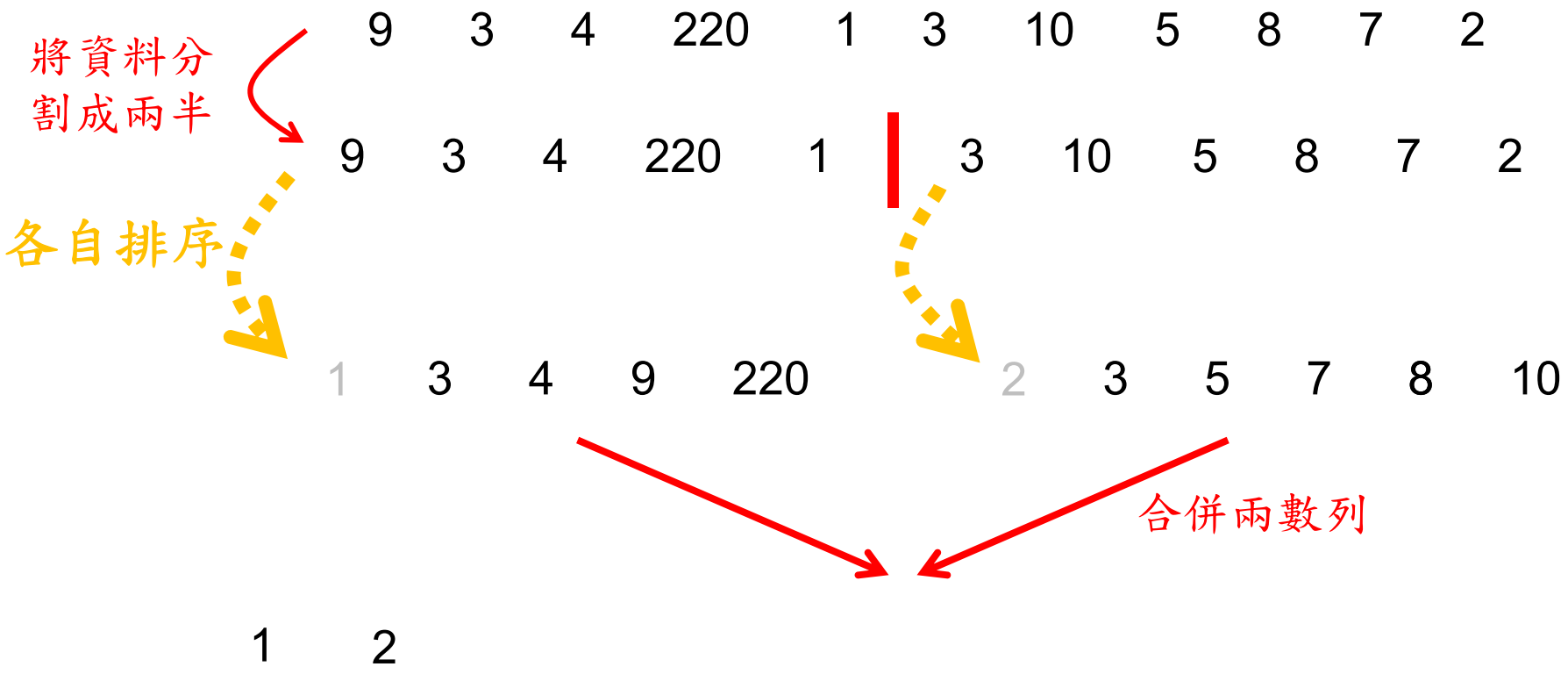

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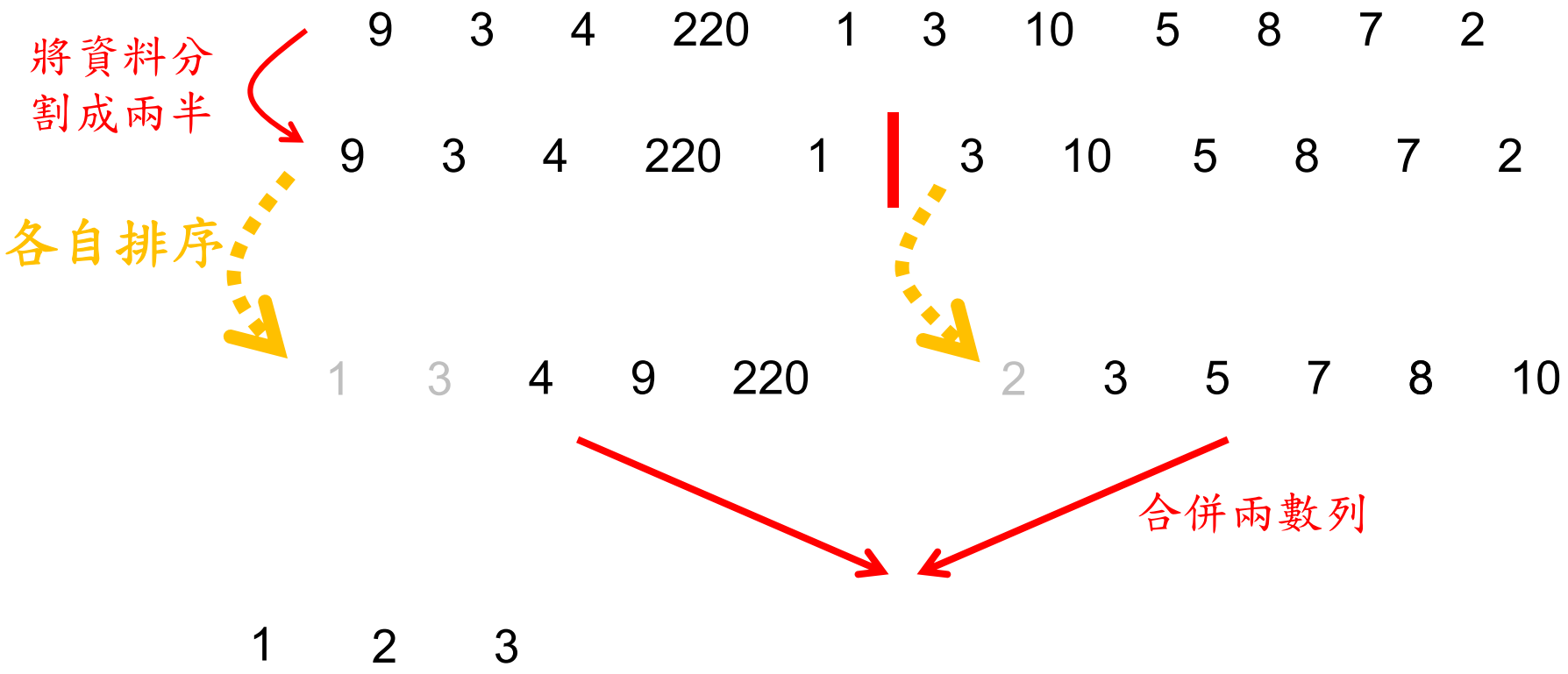

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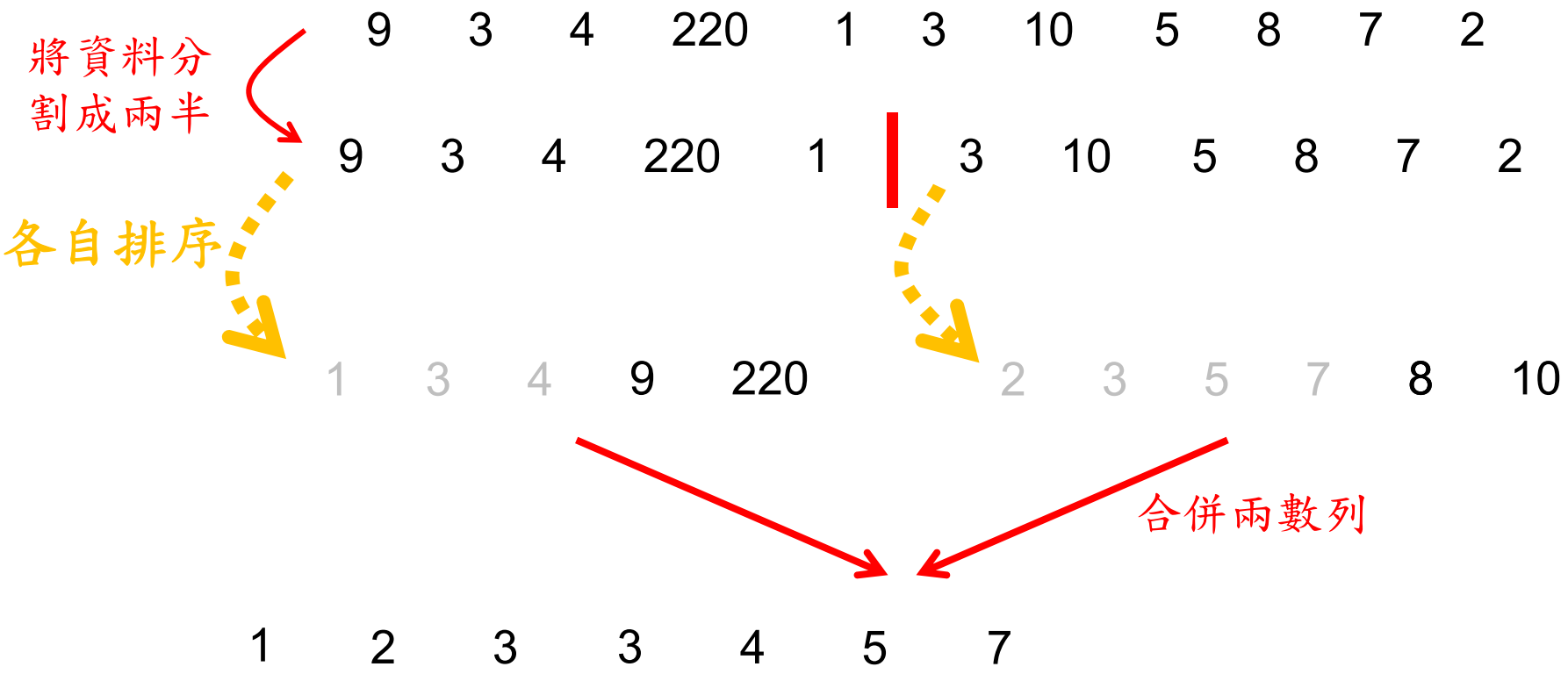

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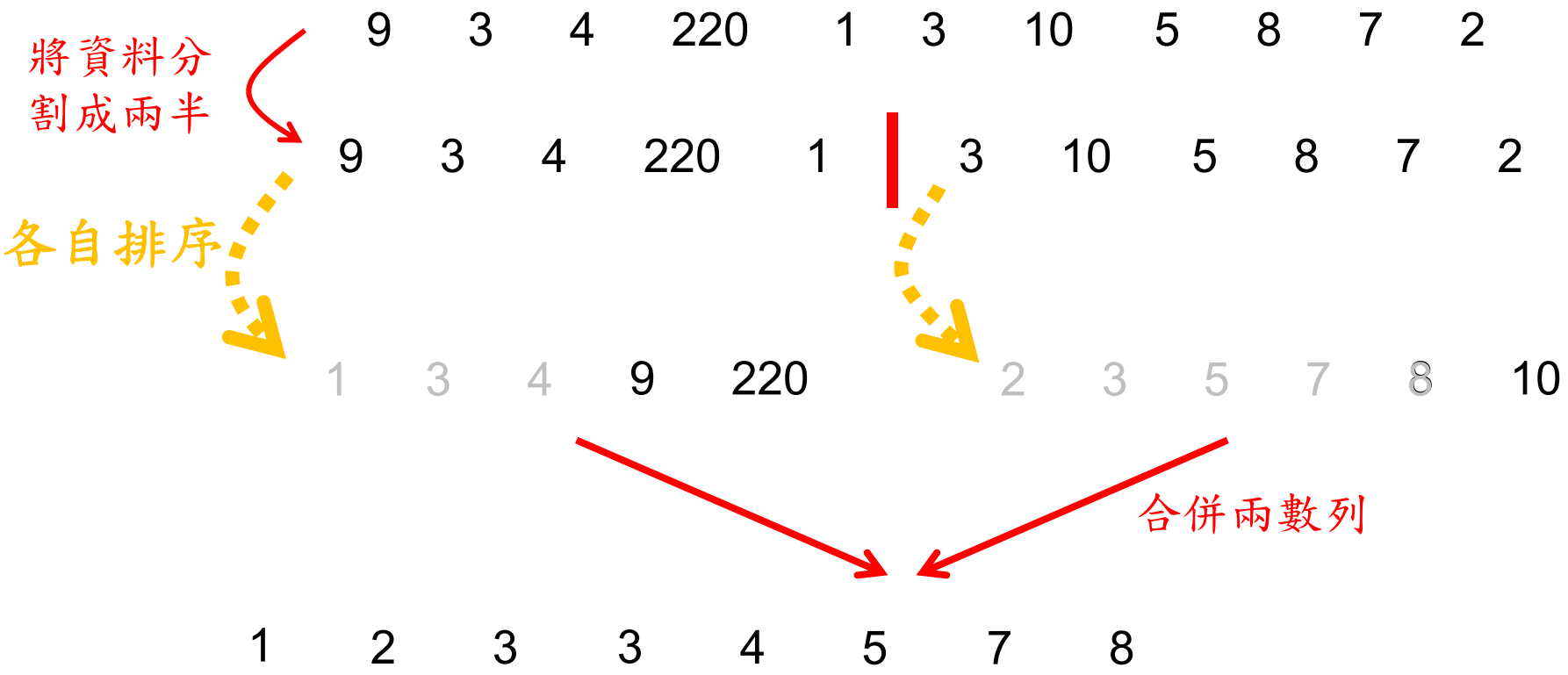

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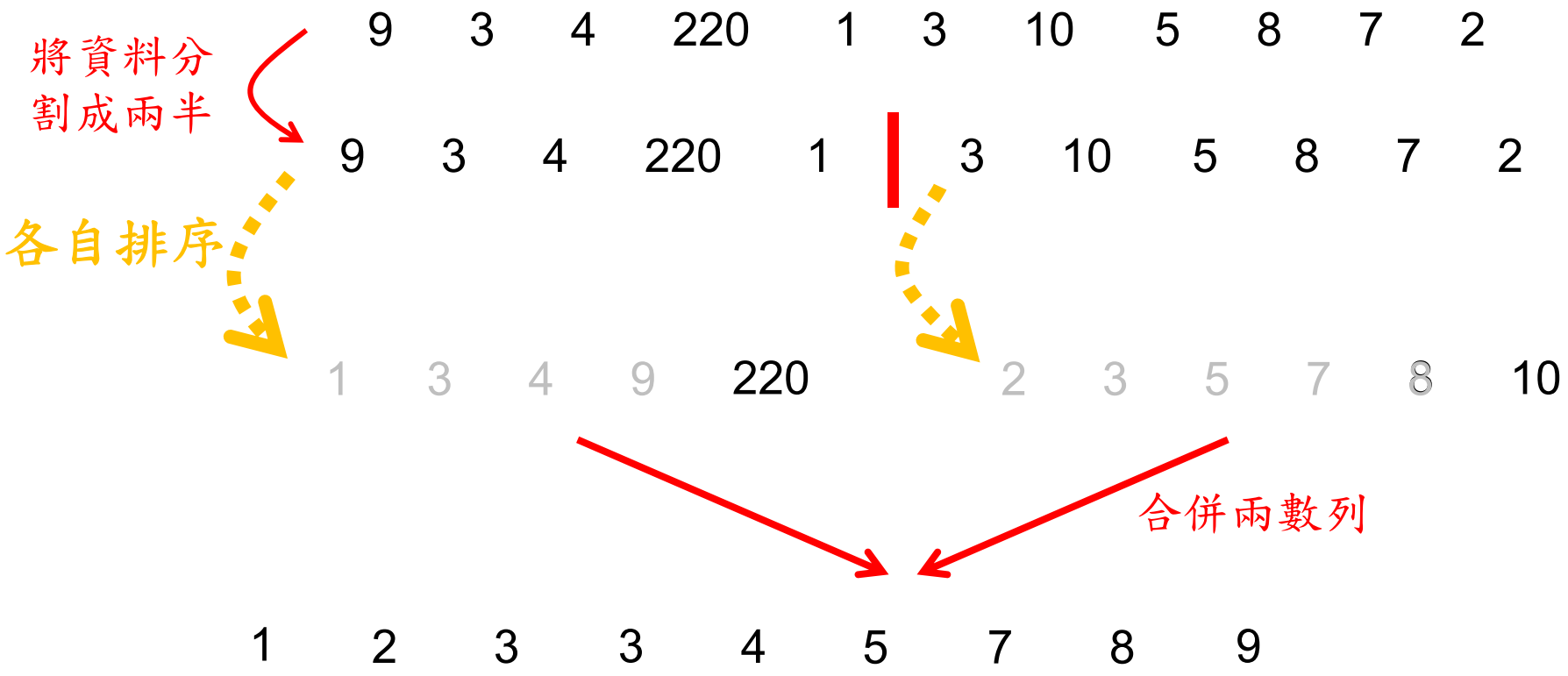

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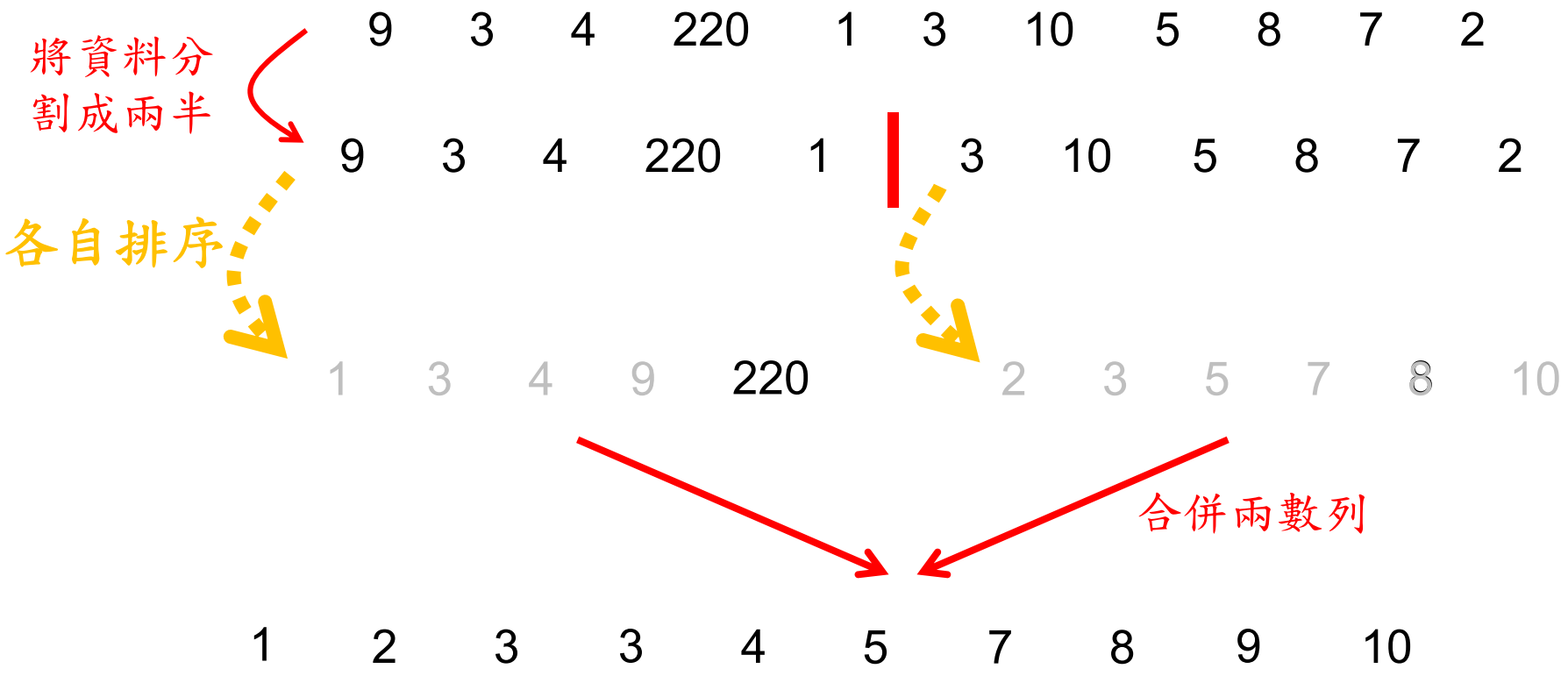

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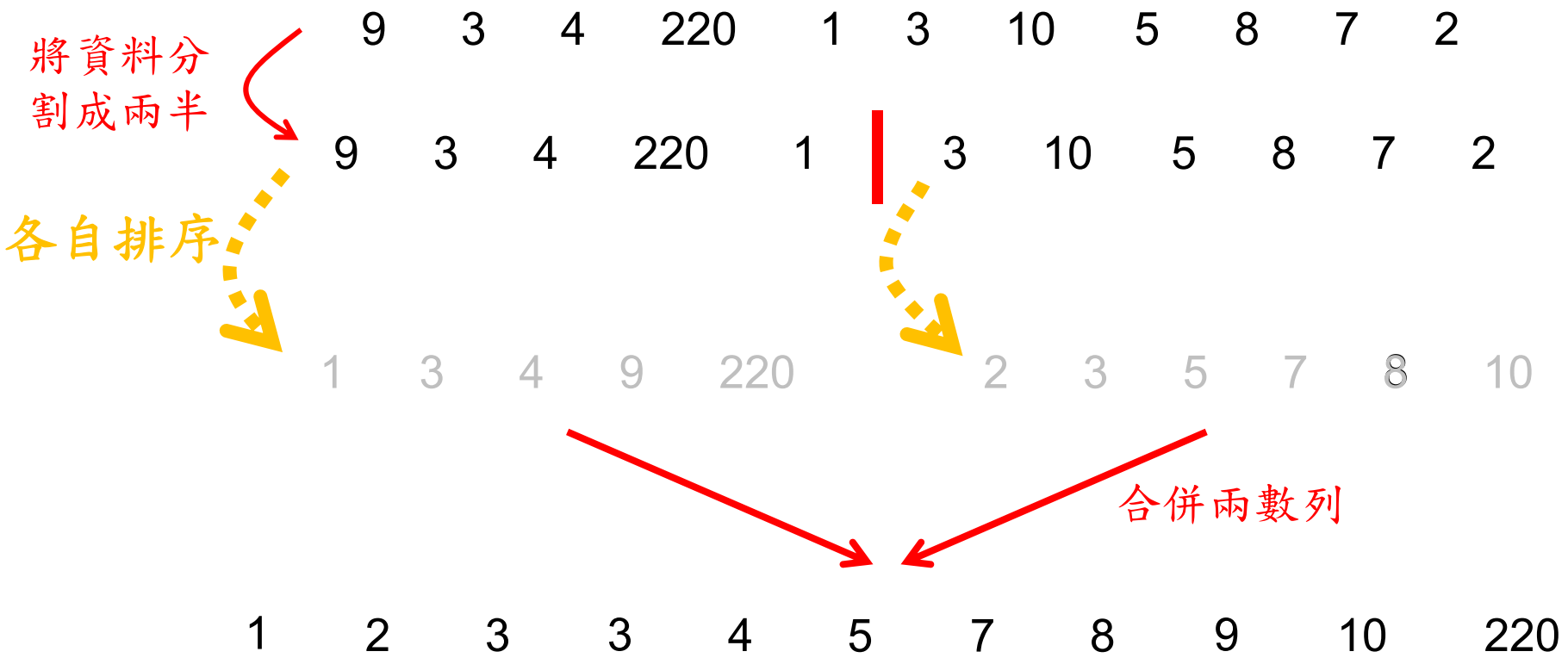

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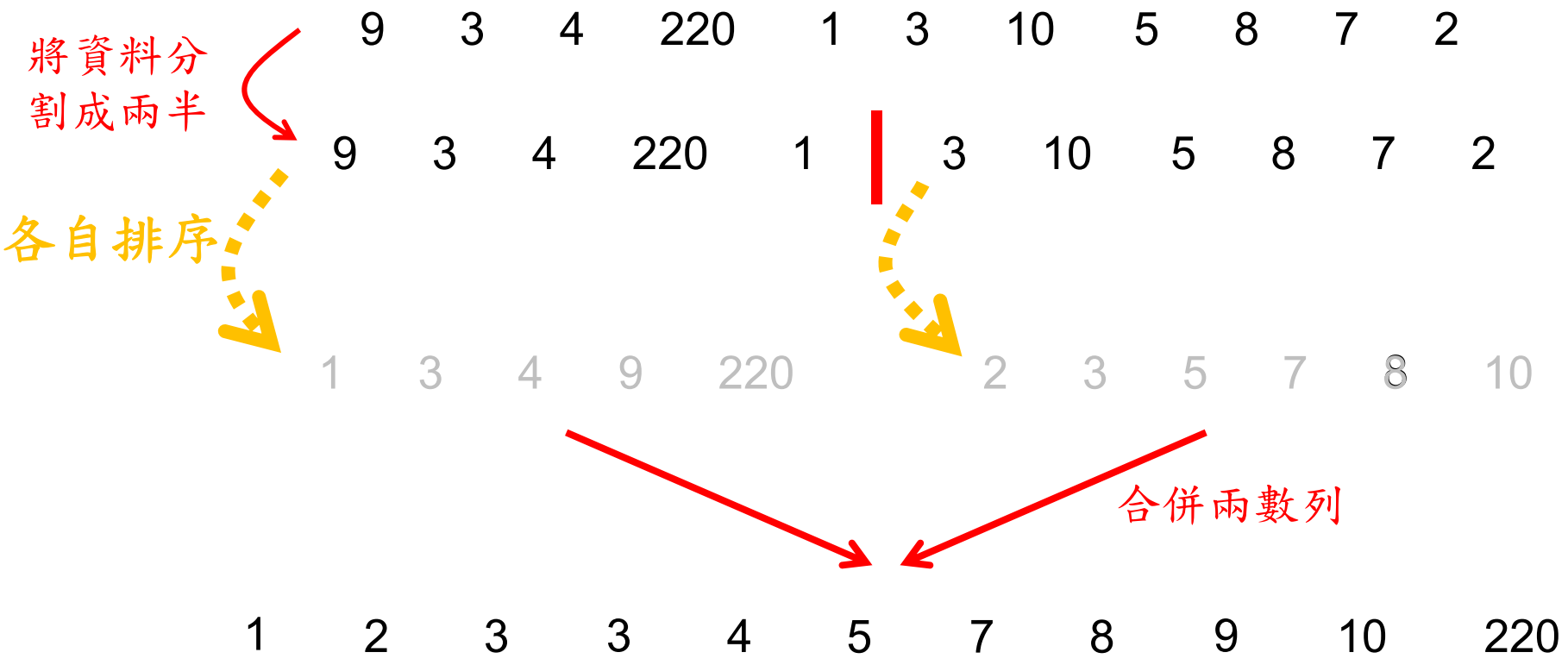

- 運算複雜度: O(n log n), 比 quicksort 的平均 O(n log n) 好
- 需要額外的 O(n) 記憶體, 比 quicksort 多

```
void mergesort(int data[], int len) {
    int len1=len/2, len2=len-len1;
    if (len <= 1) return;
    mergesort(data, len1);  mergesort(&data[len1], len2);
    merge(data, len1, &data[len1], len2);
}
```

```
void merge(int data1[], int len1, int data2[], int len2) {
    int data1_idx=0, data2_idx=0, result_idx=0;
    int *buf = (int *) malloc((len1+len2) * sizeof(int));
    while (data1_idx < len1 || data2_idx < len2) {
        if (data1_idx < len1 && data2_idx < len2)
            buf[result_idx++] = (data1[data1_idx]<=data2[data2_idx]) ?
                                        data1[data1_idx++] : data2[data2_idx++];
        else if (data1_idx < len1)
            buf[result_idx++] = data1[data1_idx++];
        else
            buf[result_idx++] = data2[data2_idx++];
    }
    for (result_idx = 0; result_idx < len1+len2; result_idx++)
        data1[result_idx] = buf[result_idx];
    free(buf);
}
```

Ugly Number

- 一個正整數如果質因數只有 2 或是 3 或是 5 的時候稱為 Ugly number;一個正整數如果質因數只有 2, 3, 5, 7 時稱為 Humble number

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```
while (n%2==0) n /= 2;
if (n==1)
    printf("n is a power of 2\n");
else
    printf("n is not a power of 2\n");
```

尋找第 n 個 Ugly Number

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int n, count = 1, i = 2;
scanf("%d", &n);
while (count < n)
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  2. n 值變大的時候, 程式執行時間很久

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  - 由Ugly number $x=2^a3^b5^c$ 的定義可以看到一種列出所有 Ugly number 的方法是列舉 (a,b,c) 的數值 (0,0,0), (0,0,1), (0,1,0), (1,0,0), (0,0,2), (0,1,1), (1,0,1), (1,1,0), (0,2,0), (2,0,0), … 不過列舉的順序不太好決定, 列舉出來的 Ugly number 也不見得是由小排到大的

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1 2

4

3 6

5 10

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1 2 3

4 6

6 9

5 10 15

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1 2 3 4

6 8

6 9 12

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1 2 3 4 5 6 8 9

10 12 16 18

12 15 18 24 27

10 15 20 25 30 40 45

尋找第 n 個 Ugly Number

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1 2 3 4 5 6 8 9 10

12 16 18 20

12 15 18 24 27 30

10 15 20 25 30 40 45 50

尋找第 n 個 Ugly Number

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1 2 3 4 5 6 8 9 10 12

16 18 20 24

12 15 18 24 27 30 36

15 20 25 30 40 45 50 60

尋找第 n 個 Ugly Number

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16 18 20 24

15 18 24 27 30 36

15 20 25 30 40 45 50 60

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1 2 3 4 5 6 8 9 10 12 15

16 18 20 24 30

18 24 27 30 36 45

15 20 25 30 40 45 50 60 75

尋找第 n 個 Ugly Number

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16 18 20 24 30

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1 2 3 4 5 6 8 9 10 12 15 16 …

18 20 24 30 32 …

18 24 27 30 36 45 48 …

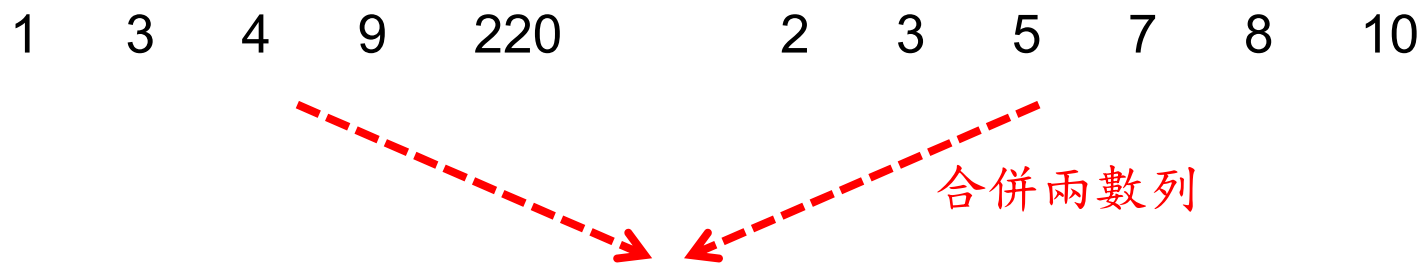
20 25 30 40 45 50 60 75 80 …

合併兩個數列

合併兩個數列

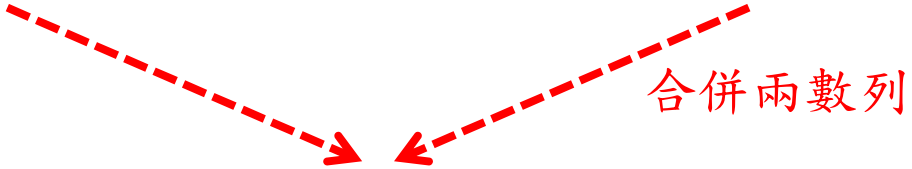
1　3　4　9　220　　2　3　5　7　8　10

合併兩個數列



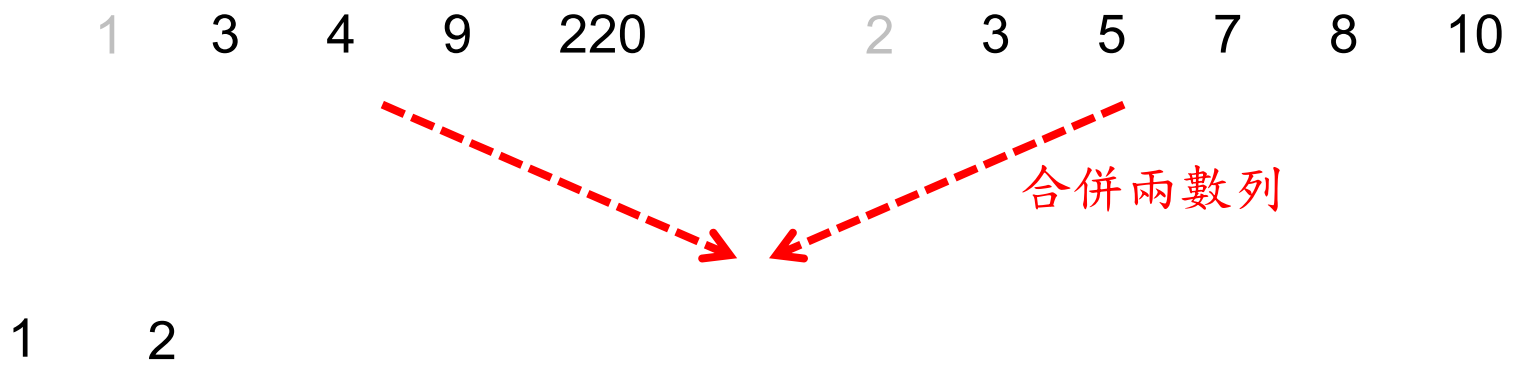
合併兩個數列

1 3 4 9 220 2 3 5 7 8 10

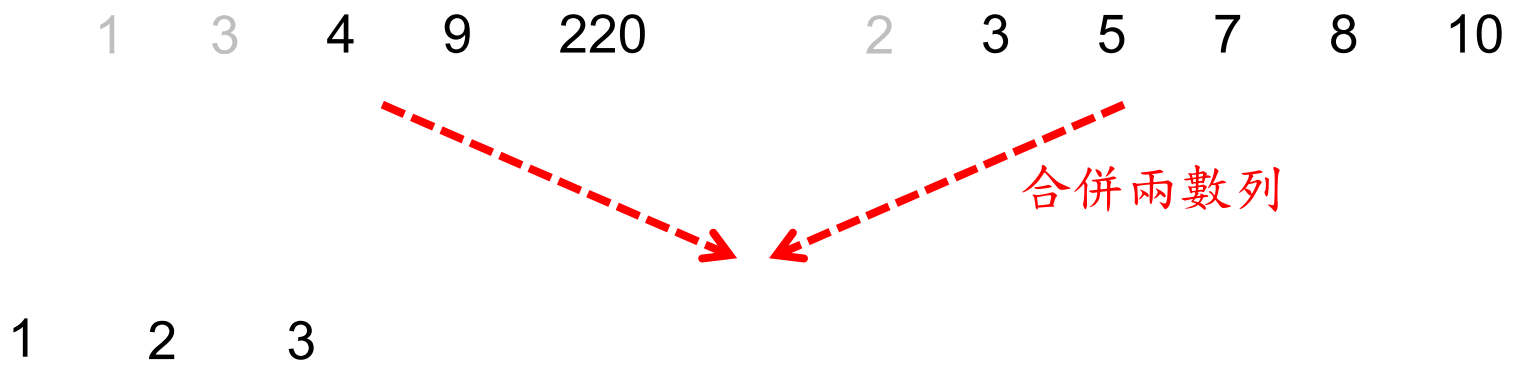


1

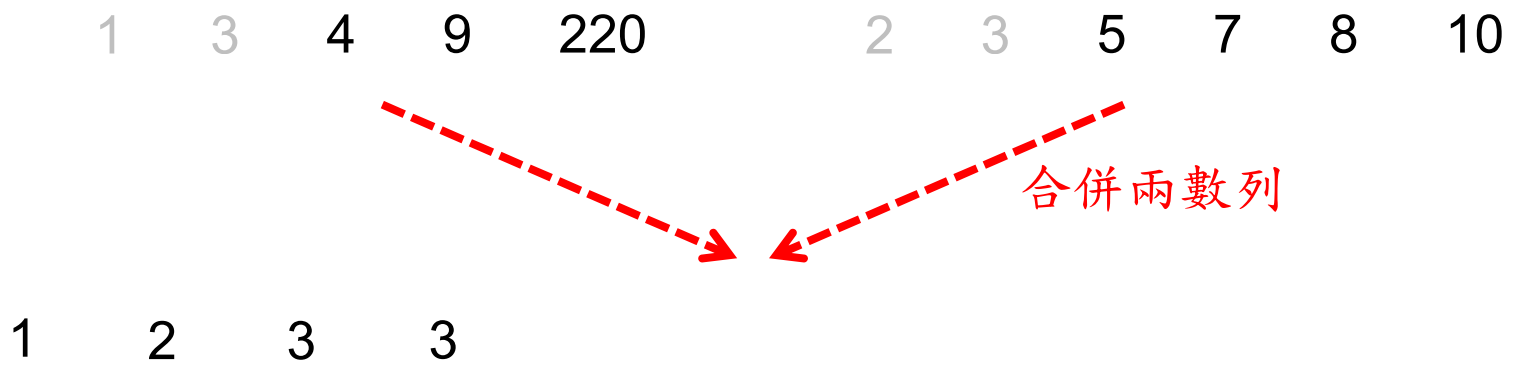
合併兩個數列



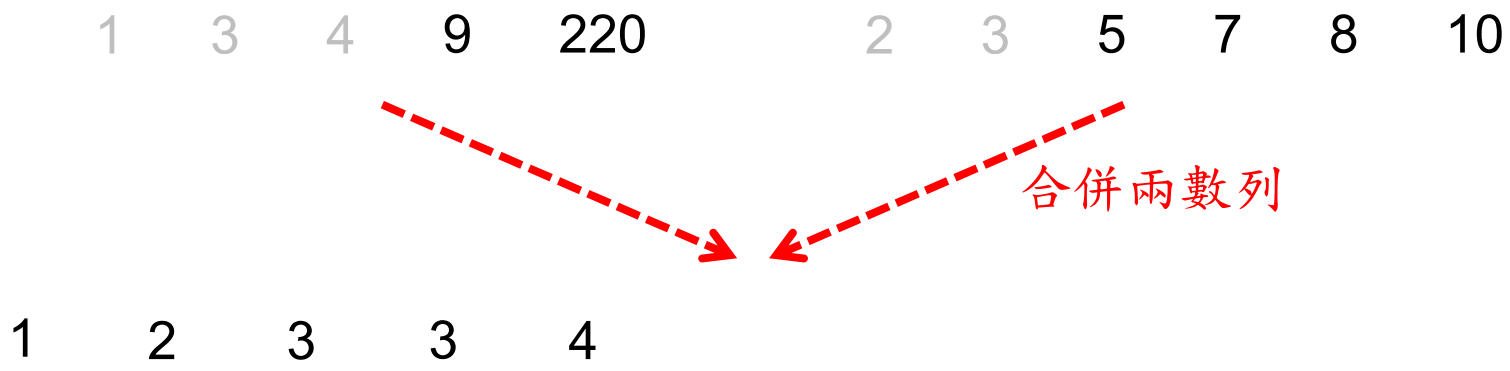
合併兩個數列



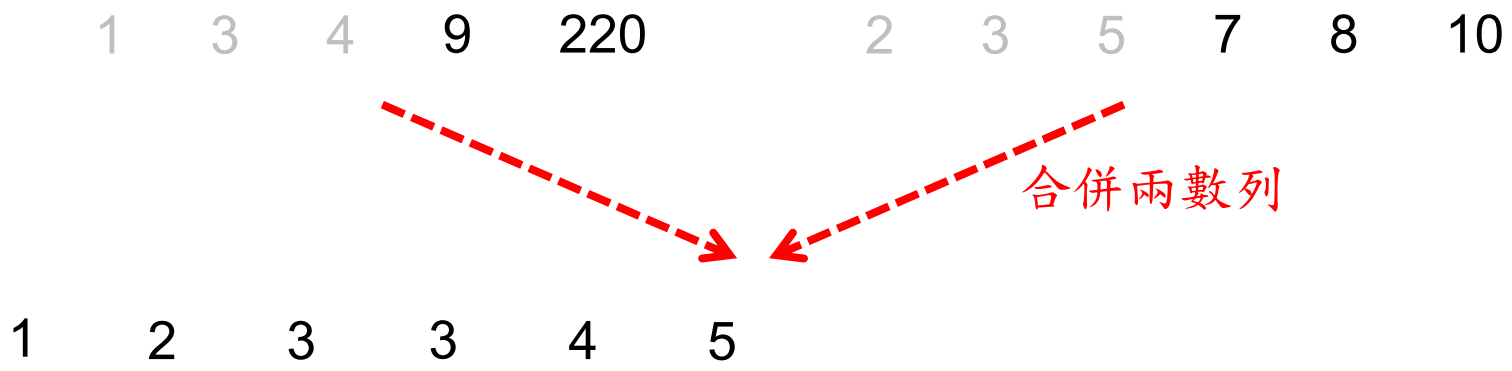
合併兩個數列



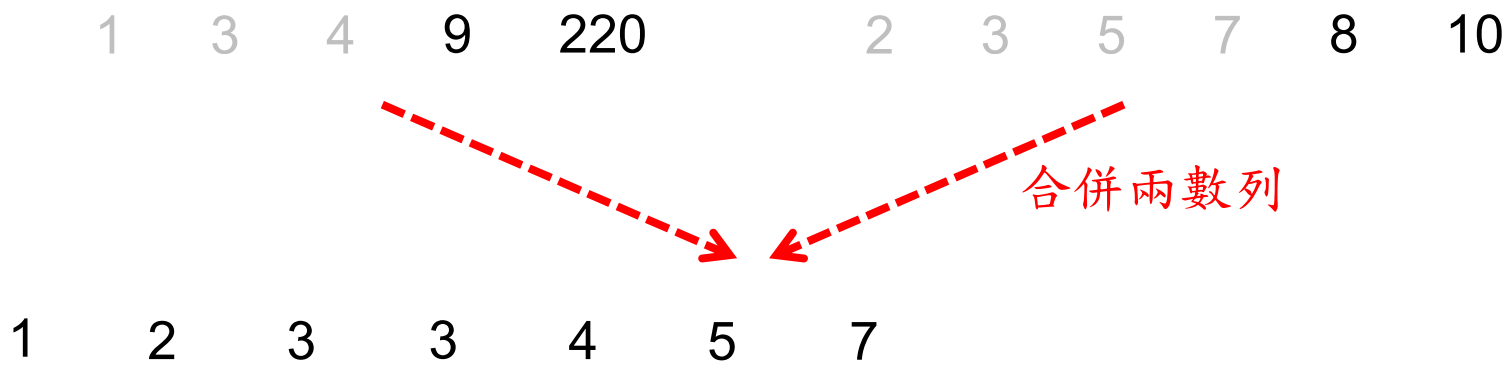
合併兩個數列



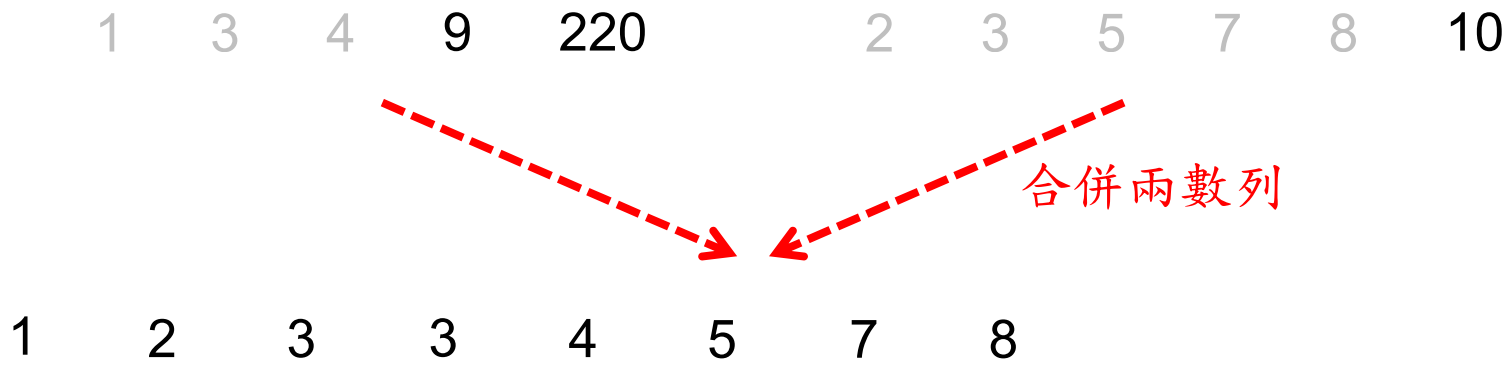
合併兩個數列



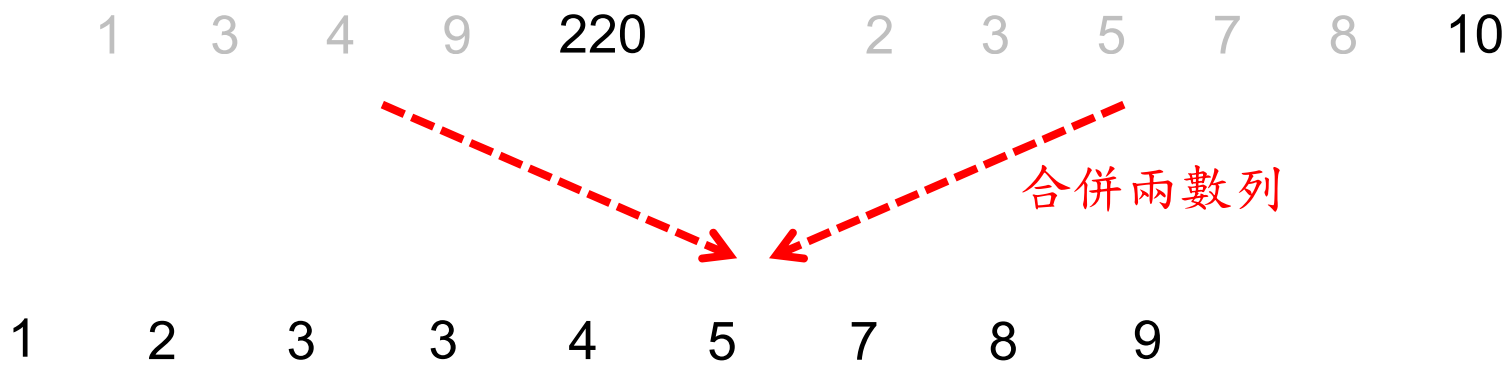
合併兩個數列



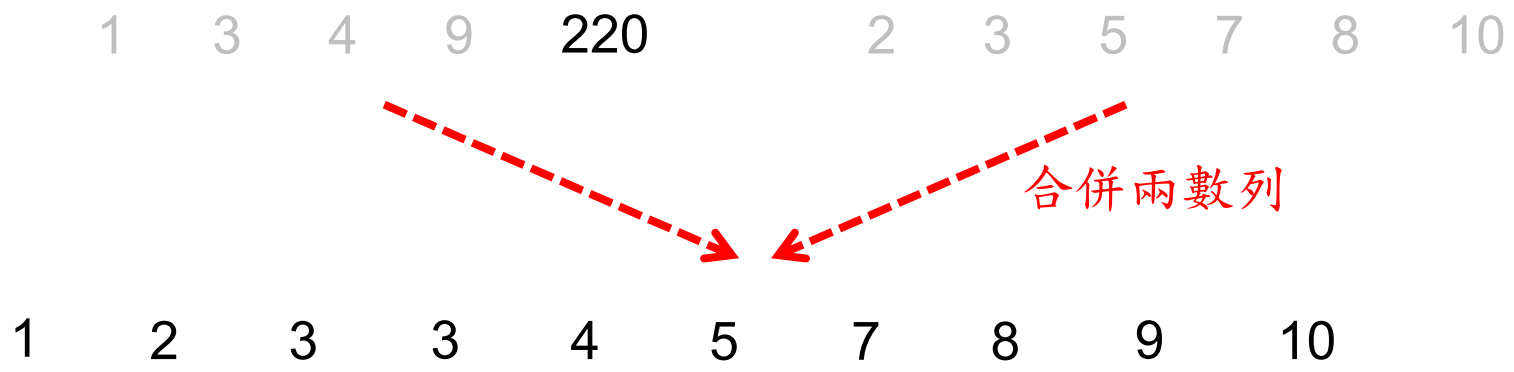
合併兩個數列



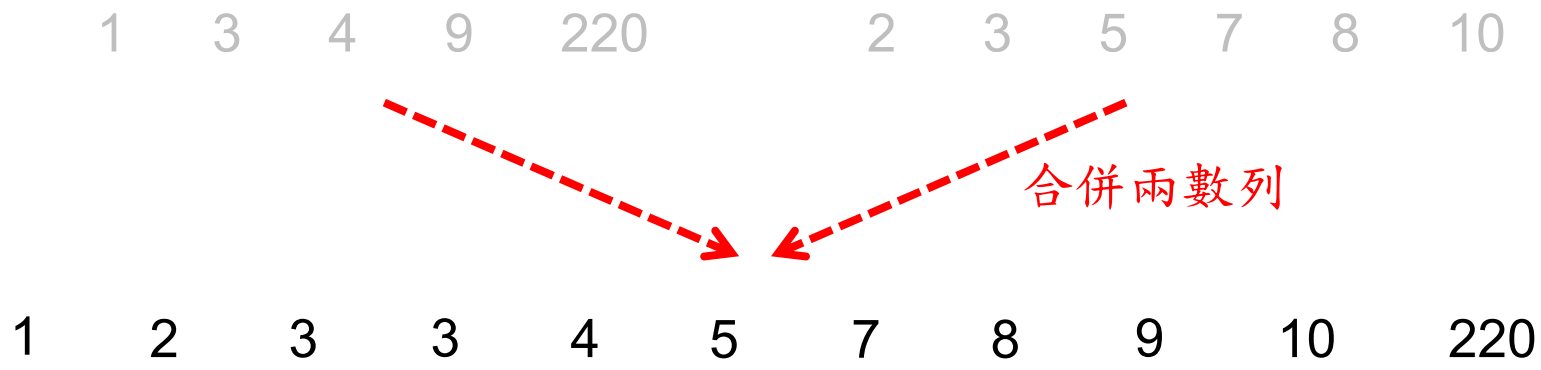
合併兩個數列



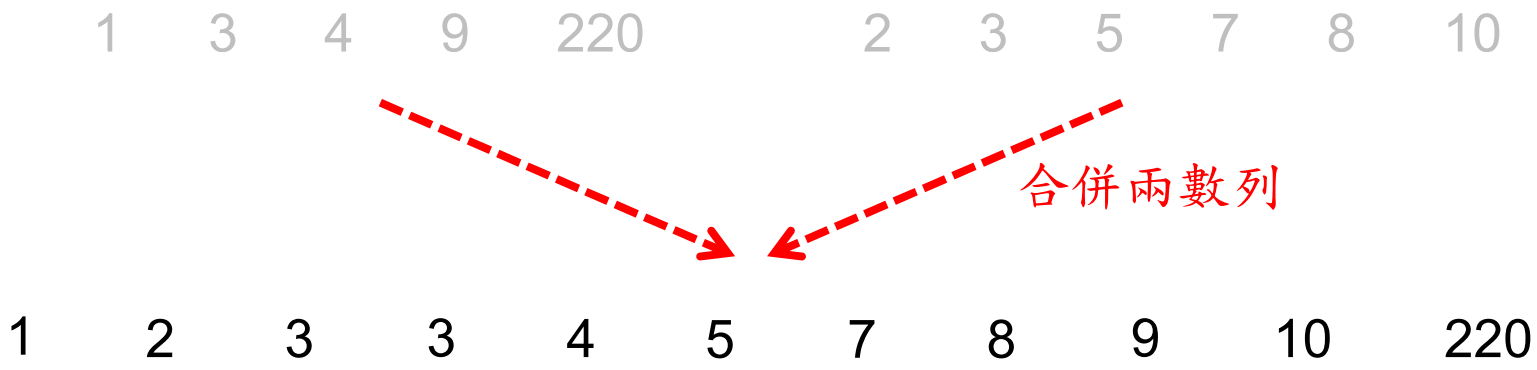
合併兩個數列



合併兩個數列



合併兩個數列

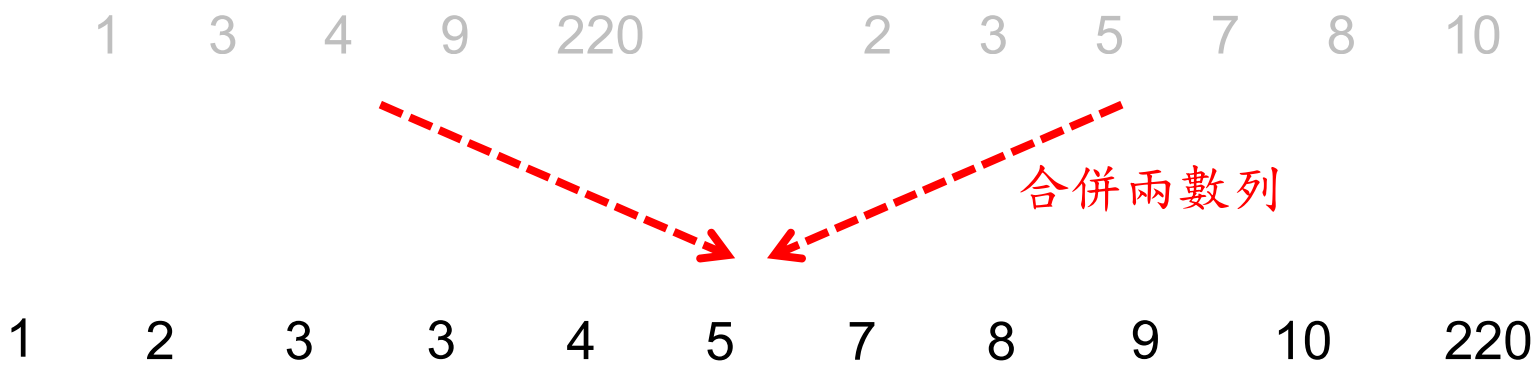



```
void merge(int data1[], int len1, int data2[], int len2, int rst[]) {
    int idx1=0, idx2=0, rst_idx=0;
    while (idx1 < len1 || idx2 < len2) {




    }
}
```

合併兩個數列

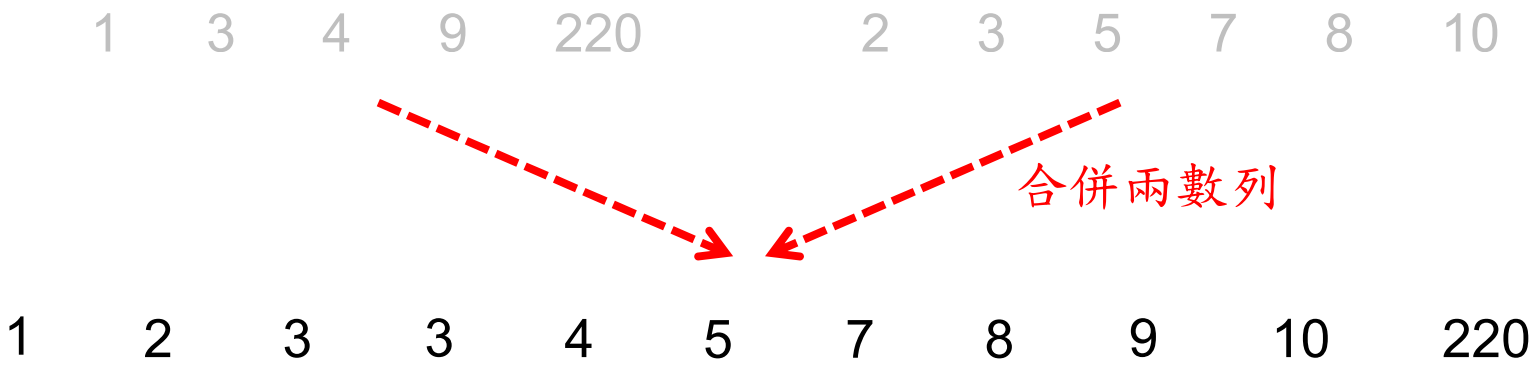



```
void merge(int data1[], int len1, int data2[], int len2, int rst[]) {
    int idx1=0, idx2=0, rst_idx=0;
    while (idx1 < len1 || idx2 < len2) {
        if (idx1 < len1 && idx2 < len2)
            rst[rst_idx++] = (data1[idx1]<=data2[idx2]) ? data1[idx1++] : data2[idx2++];




    }
}
```

合併兩個數列

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    int idx1=0, idx2=0, rst_idx=0;
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            rst[rst_idx++] = (data1[idx1]<=data2[idx2]) ? data1[idx1++] : data2[idx2++];
        else if (idx1 < len1)
            rst[rst_idx++] = data1[idx1++];
        else
            rst[rst_idx++] = data2[idx2++];
    }
}
```

尋找第 n 個 Ugly Number

- 需要以陣列紀錄接下來有哪些可能性, 由其中找到下一個 Ugly number, 因為若 a<b 則 2a<2b, 3a<3b, 5a<5b, 所以讓這些數字排成三隊, 每次只需要考慮排在最前面的三個數字就可以了

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2 4 6 8 10 12 16 18 20 24 30 32 …

3 6 9 12 15 18 24 27 30 36 45 48 …

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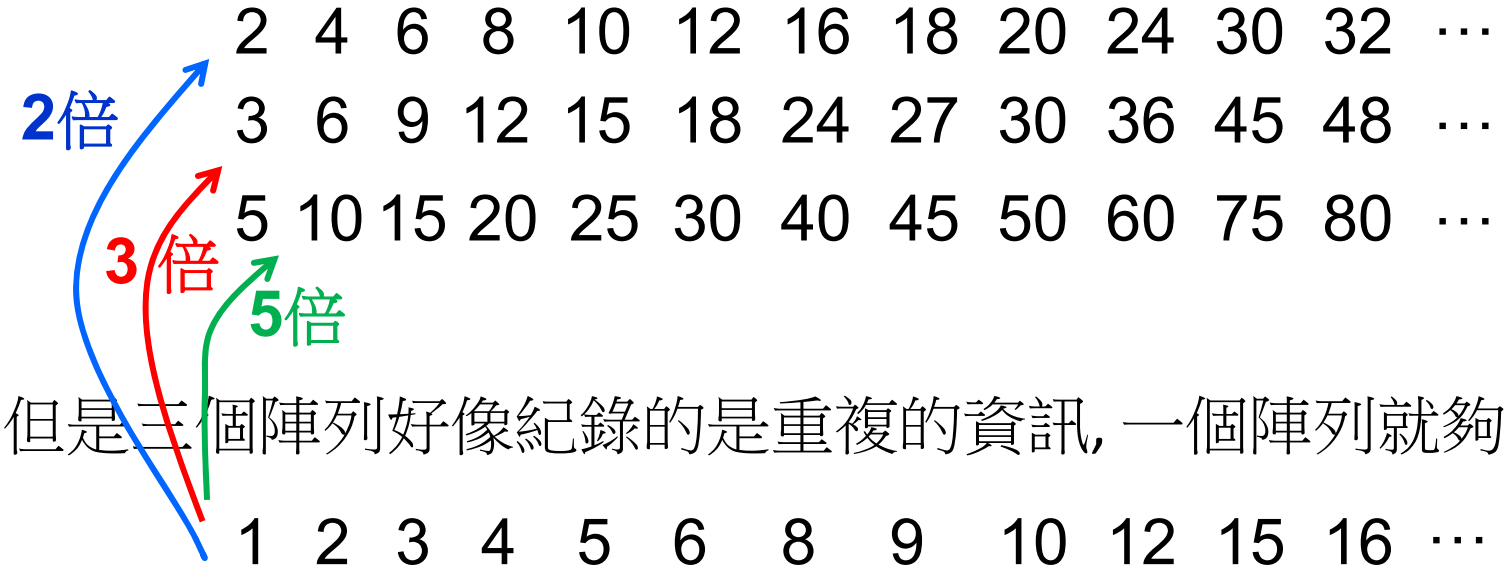


- 但是三個陣列好像紀錄的是重複的資訊, 一個陣列就夠了

1 2 3 4 5 6 8 9 10 12 15 16 …

就是所有小於 n 的 Ugly number 的陣列

尋找第 n 個 Ugly Number

尋找第 n 個 Ugly Number

```
int n, p2=1, p3=1, p5=1;
```

尋找第 n 個 Ugly Number

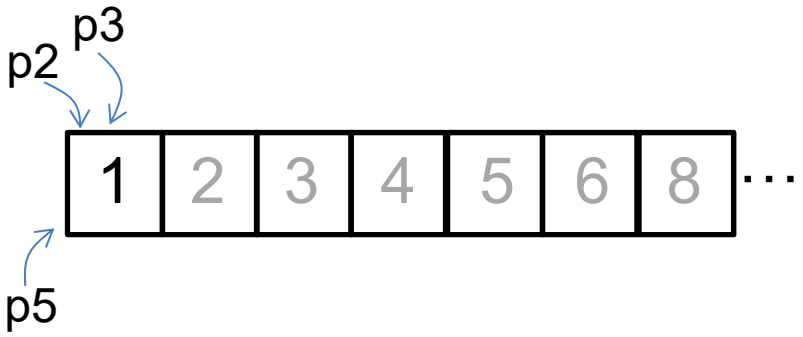
```
int n, p2=1, p3=1, p5=1;
long long int i=1, x2, x3, x5;
```

尋找第 n 個 Ugly Number

```
int n, p2=1, p3=1, p5=1;
long long int i=1, x2, x3, x5;
long long int a[10001] = {0,1};
```

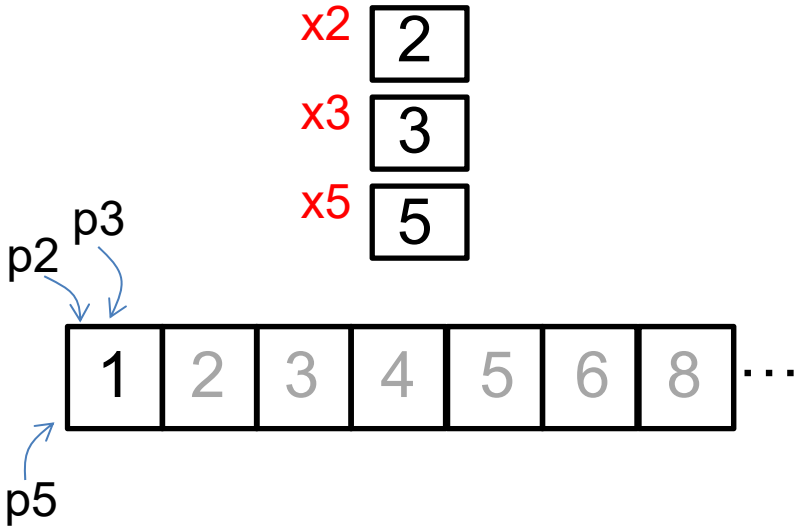
尋找第 n 個 Ugly Number

```
int n, p2=1, p3=1, p5=1;
long long int i=1, x2, x3, x5;
long long int a[10001] = {0,1};
scanf("%d", &n);
```



尋找第 n 個 Ugly Number

```
int n, p2=1, p3=1, p5=1;
long long int i=1, x2, x3, x5;
long long int a[10001] = {0,1};
scanf("%d", &n);
x2 = a[p2]*2;
x3 = a[p3]*3;
x5 = a[p5]*5;
```



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long long int i=1, x2, x3, x5;
long long int a[10001] = {0,1};
scanf("%d", &n);
x2 = a[p2]*2;
x3 = a[p3]*3;
x5 = a[p5]*5;
```
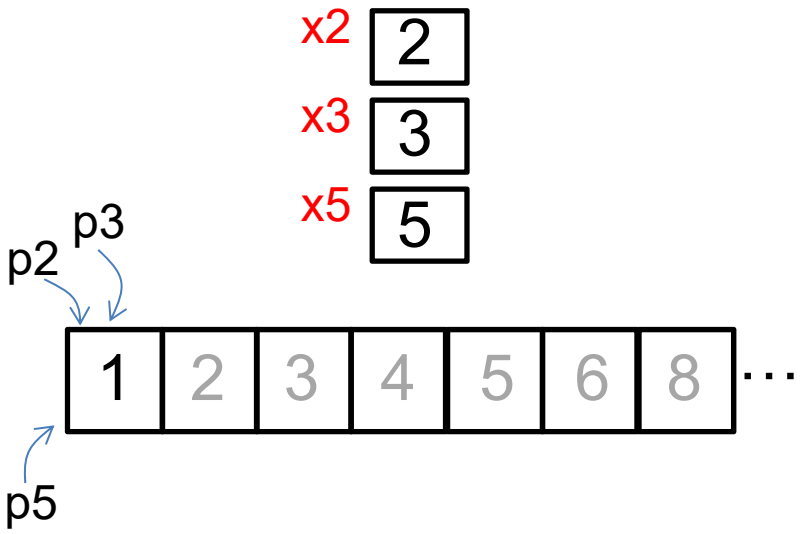
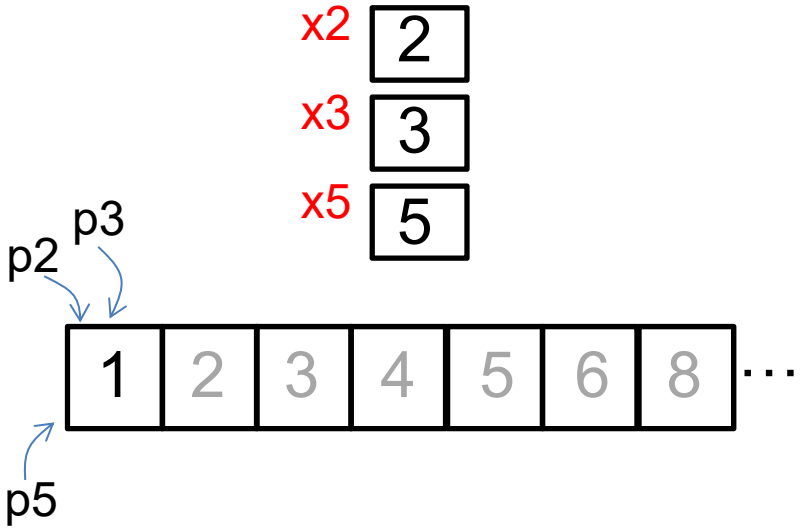




```
while (++i<=n) {
}
printf("%lld\n", a[n]);
```

尋找第 n 個 Ugly Number

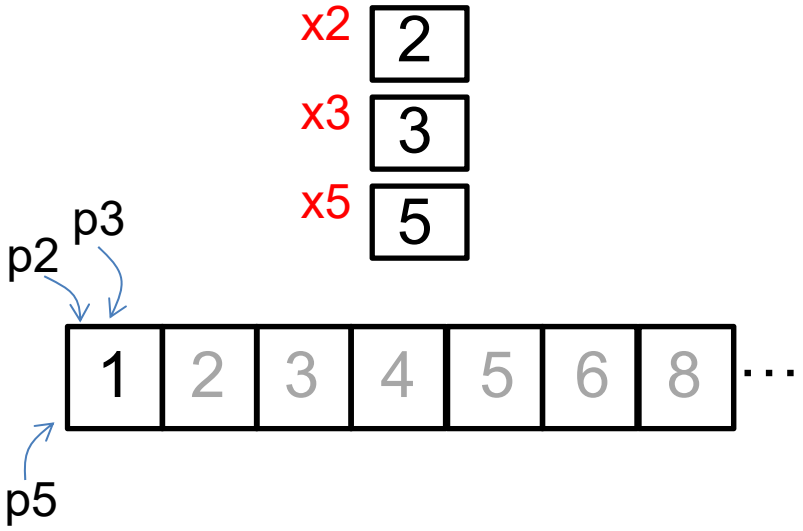
```
int n, p2=1, p3=1, p5=1;
long long int i=1, x2, x3, x5;
long long int a[10001] = {0,1};
scanf("%d", &n);
x2 = a[p2]*2;
x3 = a[p3]*3;
x5 = a[p5]*5;
```




```
while (++i<=n) {
    if (x2<x3&&x2<x5) {
        a[i] = x2; x2 = a[++p2]*2;
        while ((x2==x3) || (x2==x5))
          x2 = a[++p2]*2;
    }
}
printf("%lld\n", a[n]);
```

尋找第 n 個 Ugly Number

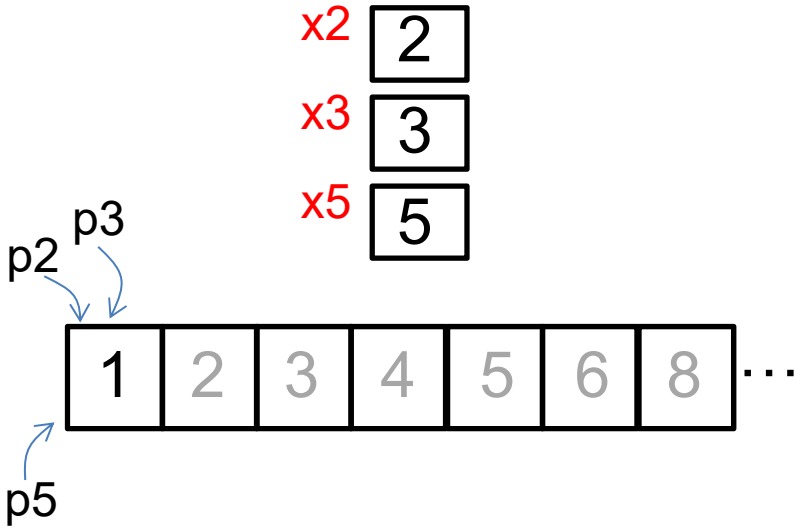
```
int n, p2=1, p3=1, p5=1;
long long int i=1, x2, x3, x5;
long long int a[10001] = {0,1};
scanf("%d", &n);
x2 = a[p2]*2;
x3 = a[p3]*3;
x5 = a[p5]*5;
```




```
while (++i<=n) {
    if (x2<x3&&x2<x5) {
        a[i] = x2; x2 = a[++p2]*2;
        while ((x2==x3) || (x2==x5))
          x2 = a[++p2]*2;
    }
    else if (x3<x2&&x3<x5) {
        a[i] = x3; x3 = a[++p3]*3;
        while ((x3==x2)||(x3==x5))
            x3 = a[++p3]*3;
    }
}
printf("%lld\n", a[n]);
```

尋找第 n 個 Ugly Number

```
int n, p2=1, p3=1, p5=1;
long long int i=1, x2, x3, x5;
long long int a[10001] = {0,1};
scanf("%d", &n);
x2 = a[p2]*2;
x3 = a[p3]*3;
x5 = a[p5]*5;
```




```
while (++i<=n) {
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        a[i] = x2; x2 = a[++p2]*2;
        while ((x2==x3) || (x2==x5))
            x2 = a[++p2]*2;
    }
    else if (x3<x2&&x3<x5) {
        a[i] = x3; x3 = a[++p3]*3;
        while ((x3==x2)||(x3==x5))
            x3 = a[++p3]*3;
    }
    else if (x5<x2&&x5<x3) {
        a[i] = x5; x5 = a[++p5]*5;
        while ((x5==x2)||(x5==x3))
            x5 = a[++p5]*5;
    }
}
printf("%lld\n", a[n]);
```

ZJ a241

有多少個 1/x 是有限小數

- 若 x 為整數且 1<x≤n，請問有幾個 1/x 是有限小數 (finite decimal)? (實數還包括循環小數 cyclic decimal 以及無理數)

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- m 位的有限小數可以寫成 $y/10^m$，其中 y 不是 10 的整數倍

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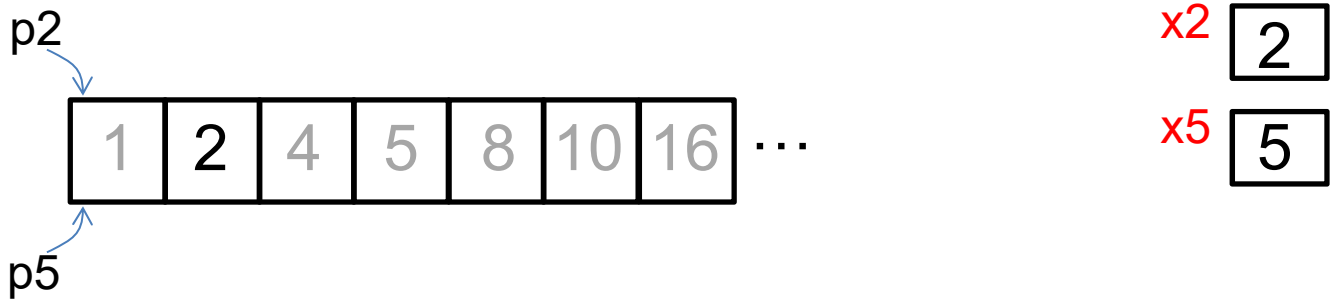
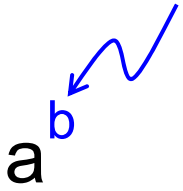
- 尋找可能的 x 基本上和尋找 Ugly Number 的方法是一樣的，數量事實上很少， $1\sim10^8$ 之間只有 171 個



LC315 Count of Smaller Numbers after Self

O(n^2) solution is trivial, the goal is to devise an O(n logn) solution.

希望得到的結果
(小於 a 的個數)



7	3	5	2	4	9	2	6
6	2	3	0	1	2	0	0

逆序, x > y
(backward sequence,
inverted sequence)

(7,3), (3,2), … 逆序對

逆序數: 14
序列中逆序對的數目

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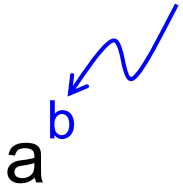

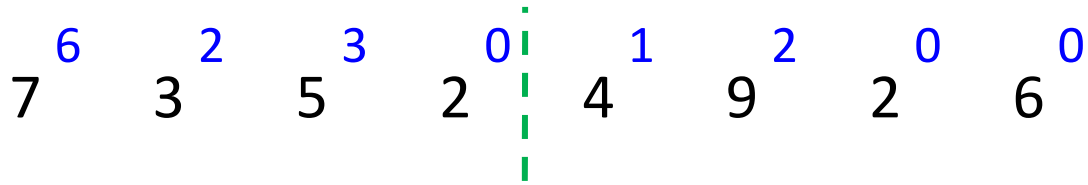

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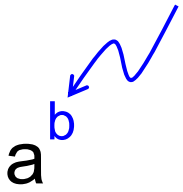

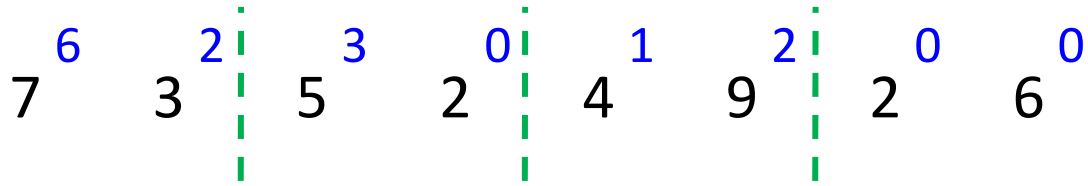

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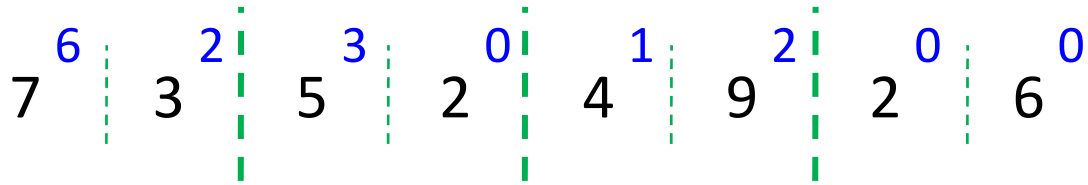
希望得到的結果
(小於 a 的個數)

a b



逆序, x > y
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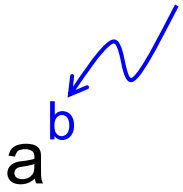

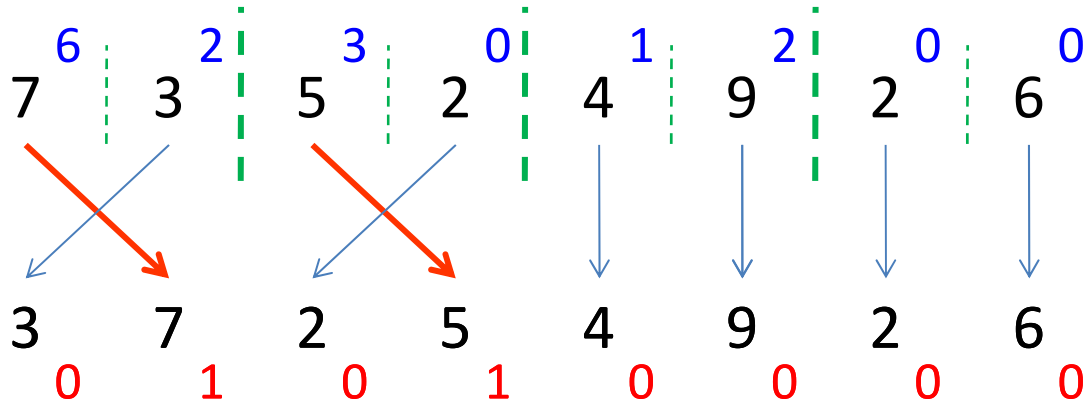

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Mergesort 過程中往右移 **k** 位的意思是表示右邊有 **k** 個比較小的數字

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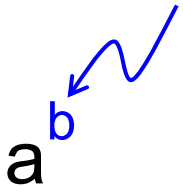

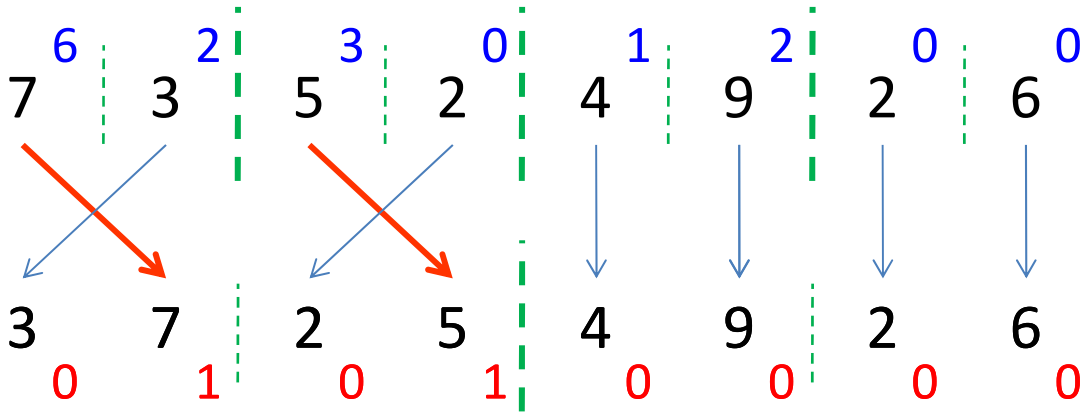

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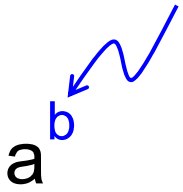

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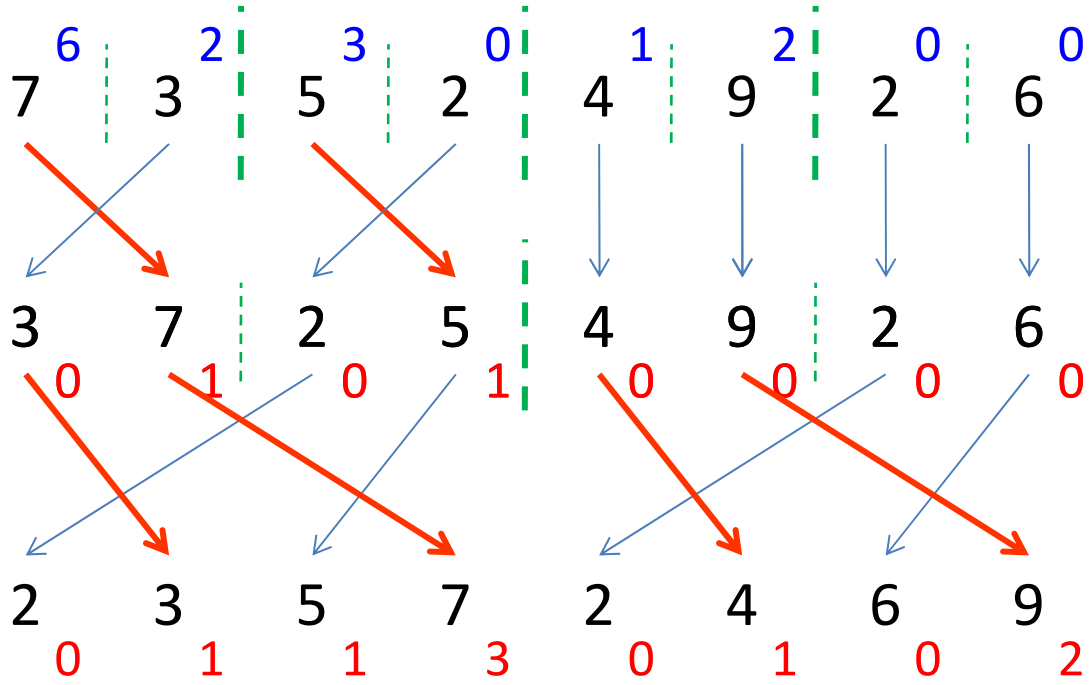

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希望得到的結果
(小於 a 的個數)

a^b

逆序, x > y
(backward sequence,
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逆序數: 14
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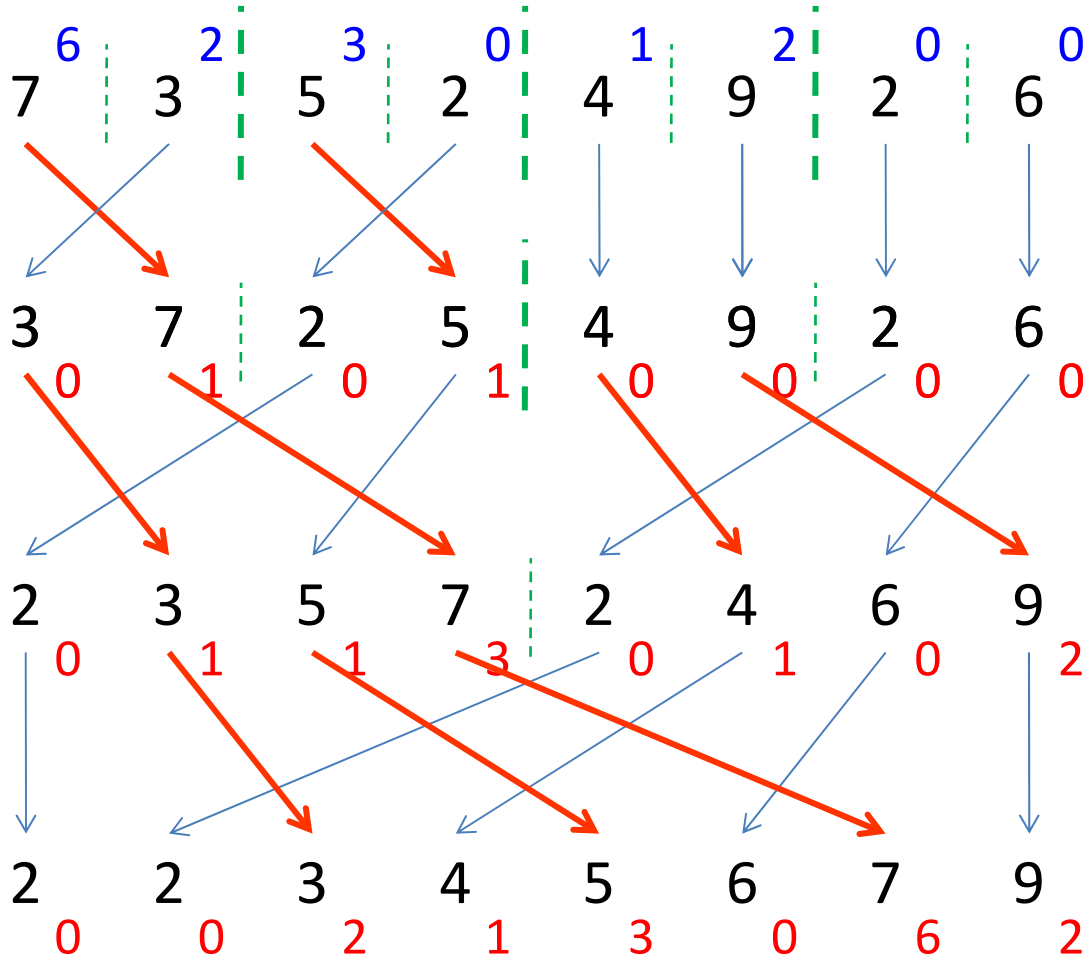

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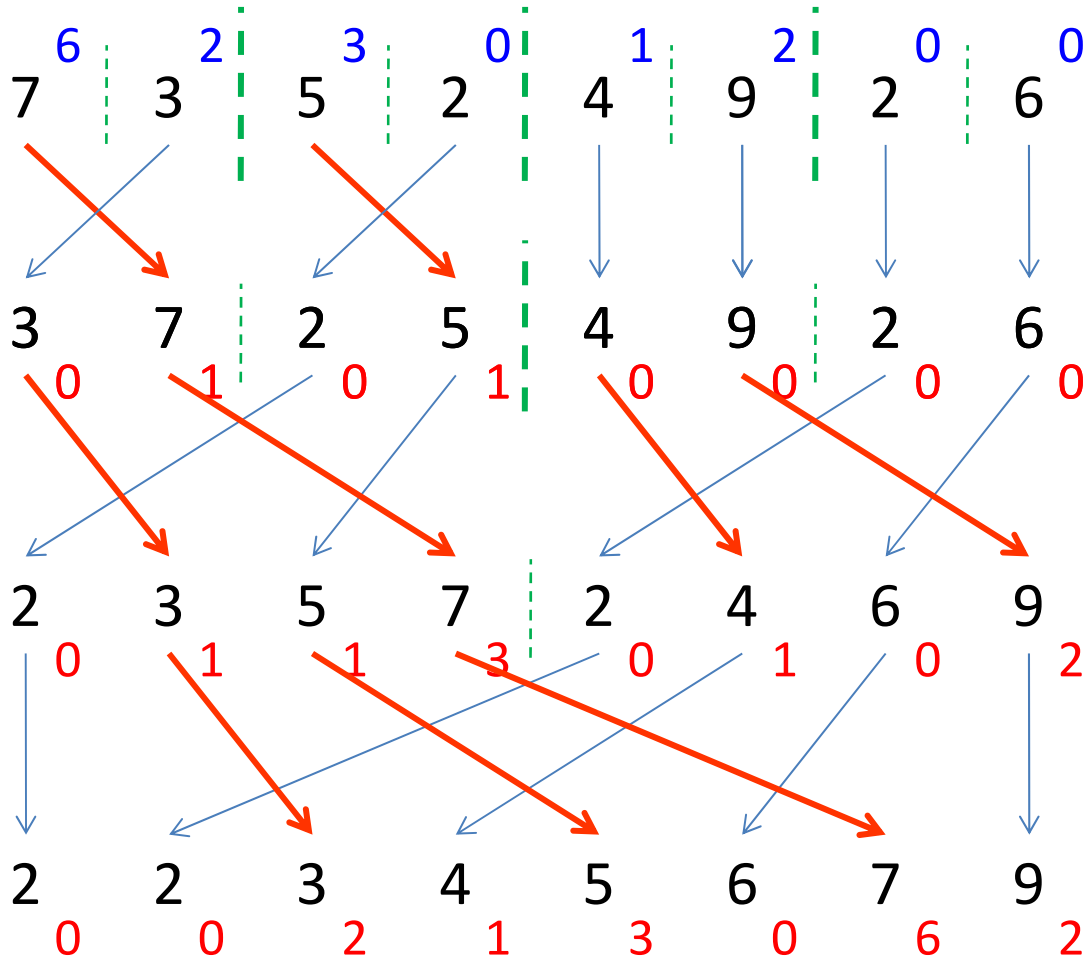
UVa10810, ZJ b684



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LC327 Count of Range Sum

3 -2 2 1 -3 -2 0 -1

LC327 Count of Range Sum

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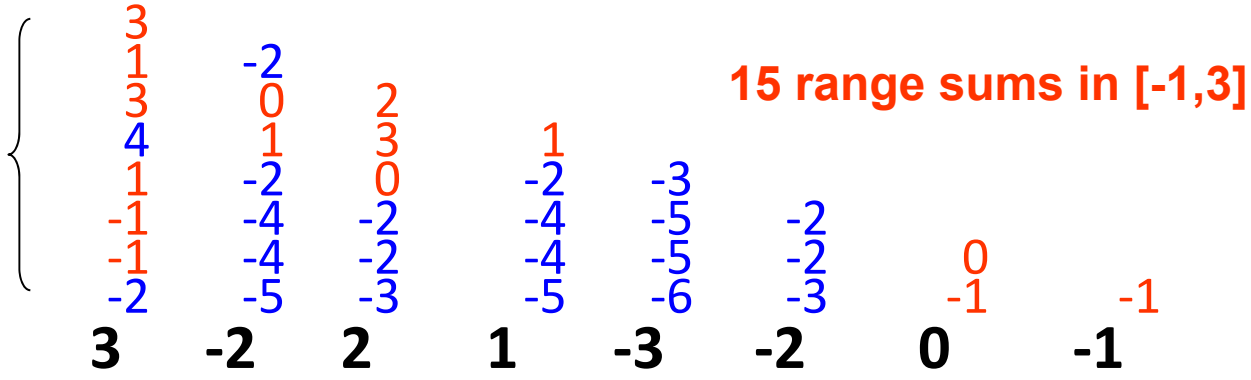

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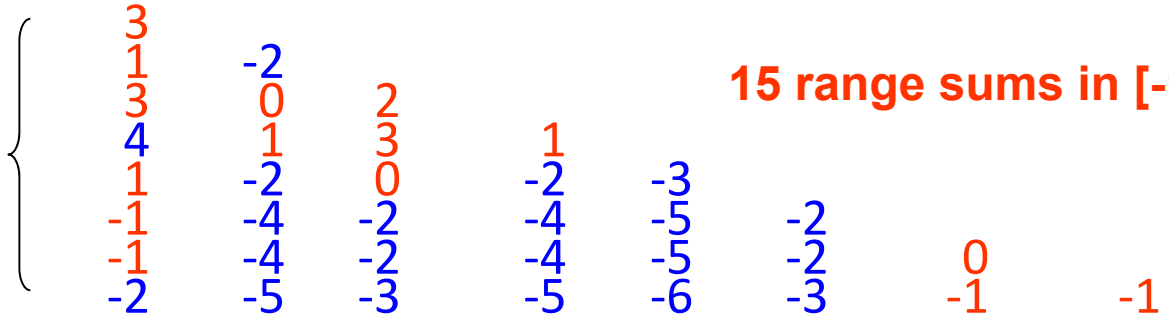

	3	**-2**	**2**	**1**	**-3**	**-2**	**0**	**-1**
0	3	1	3	4	1	-1	-1	-2

LC327 Count of Range Sum

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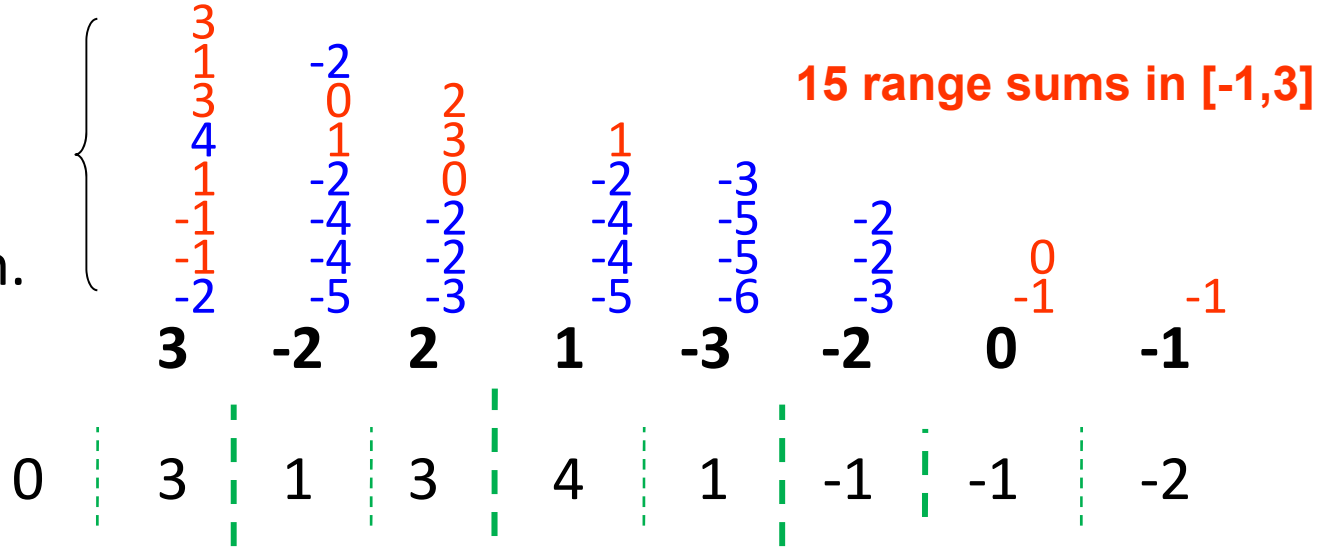

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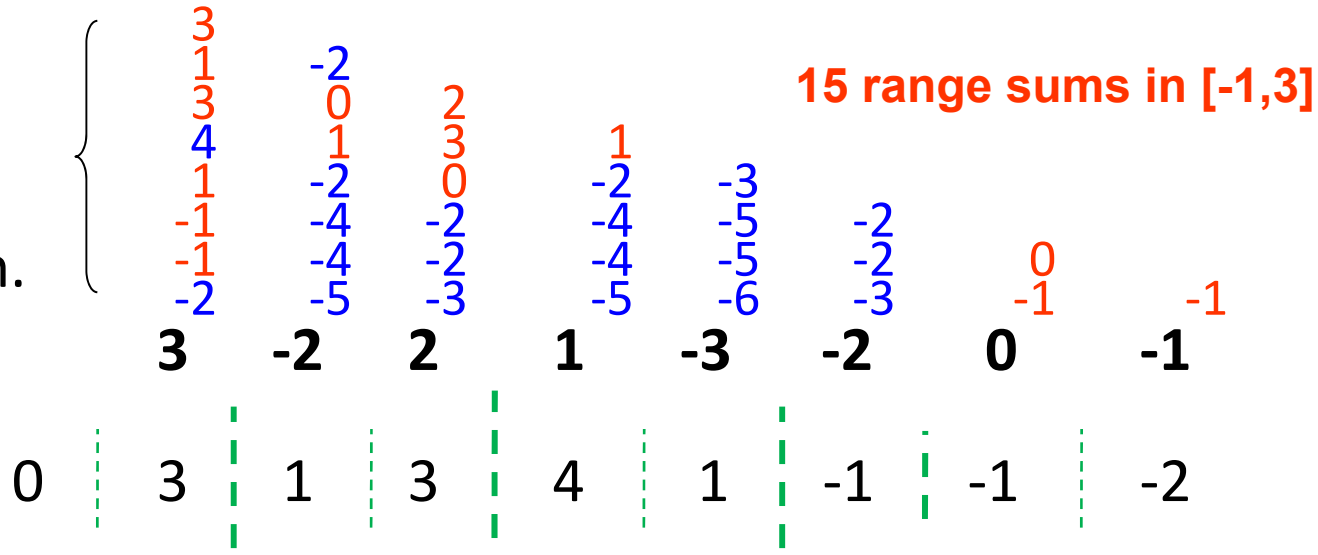

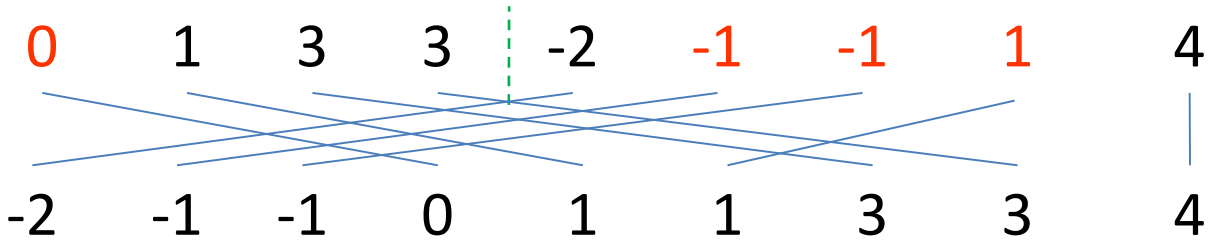

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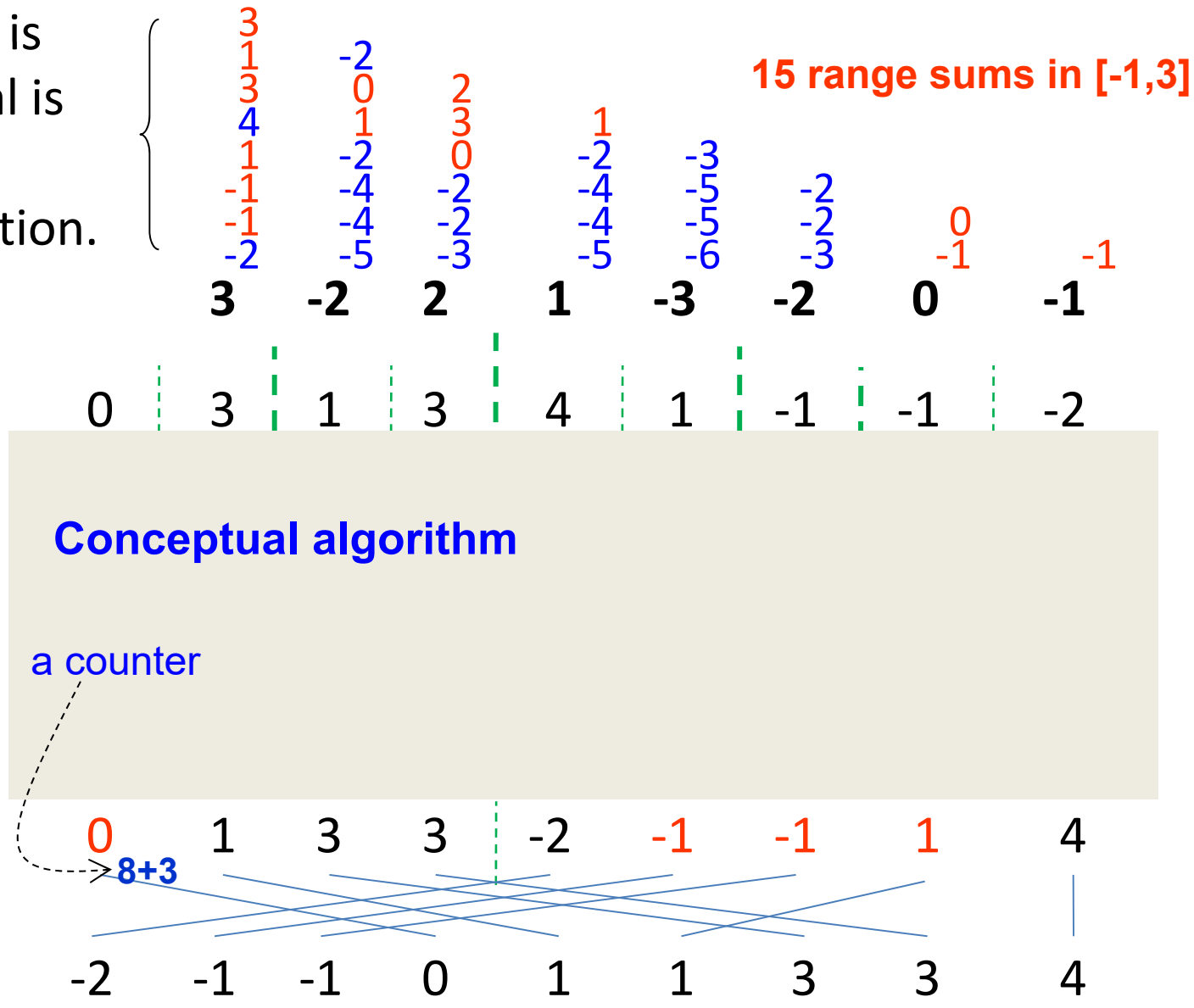

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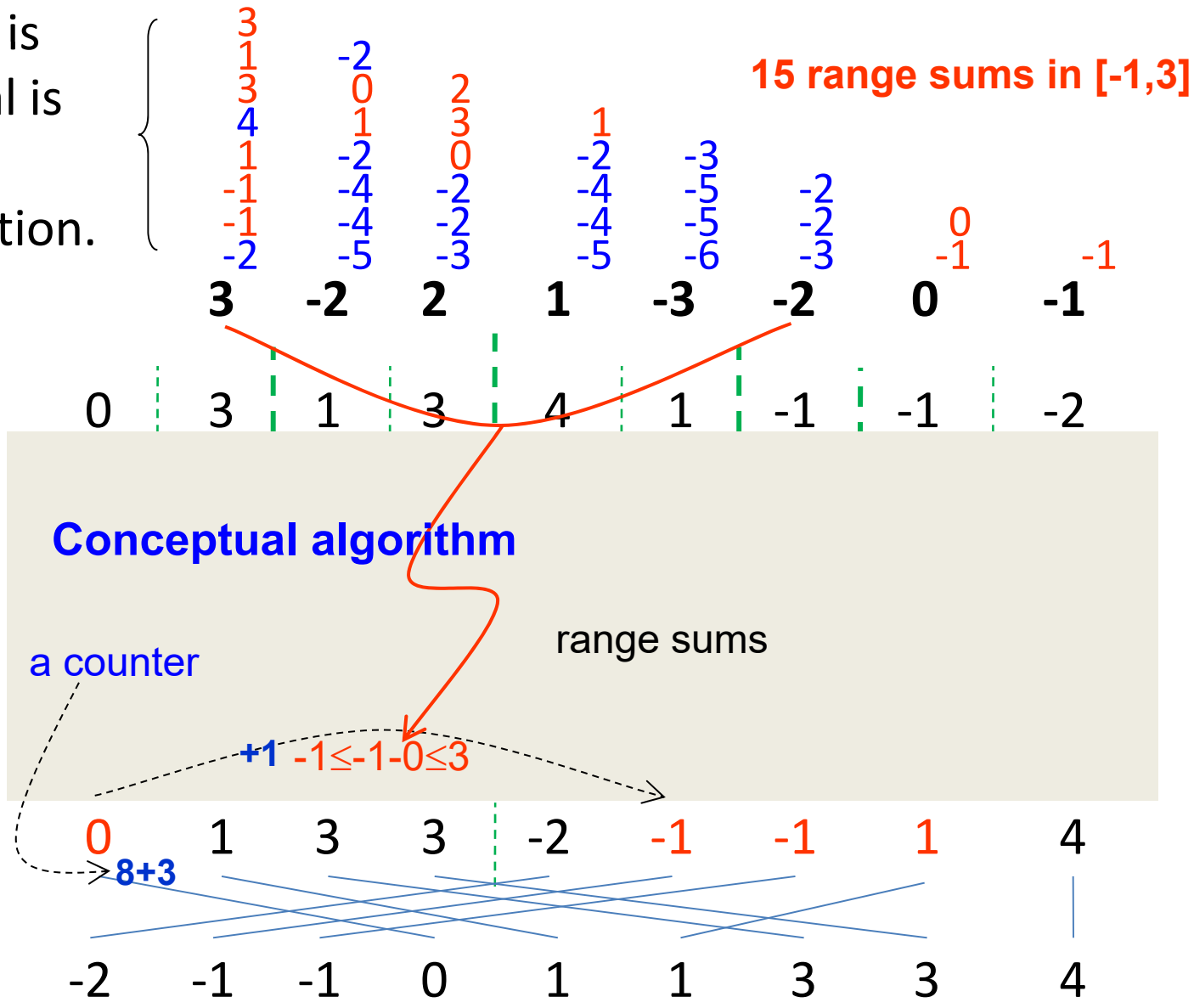

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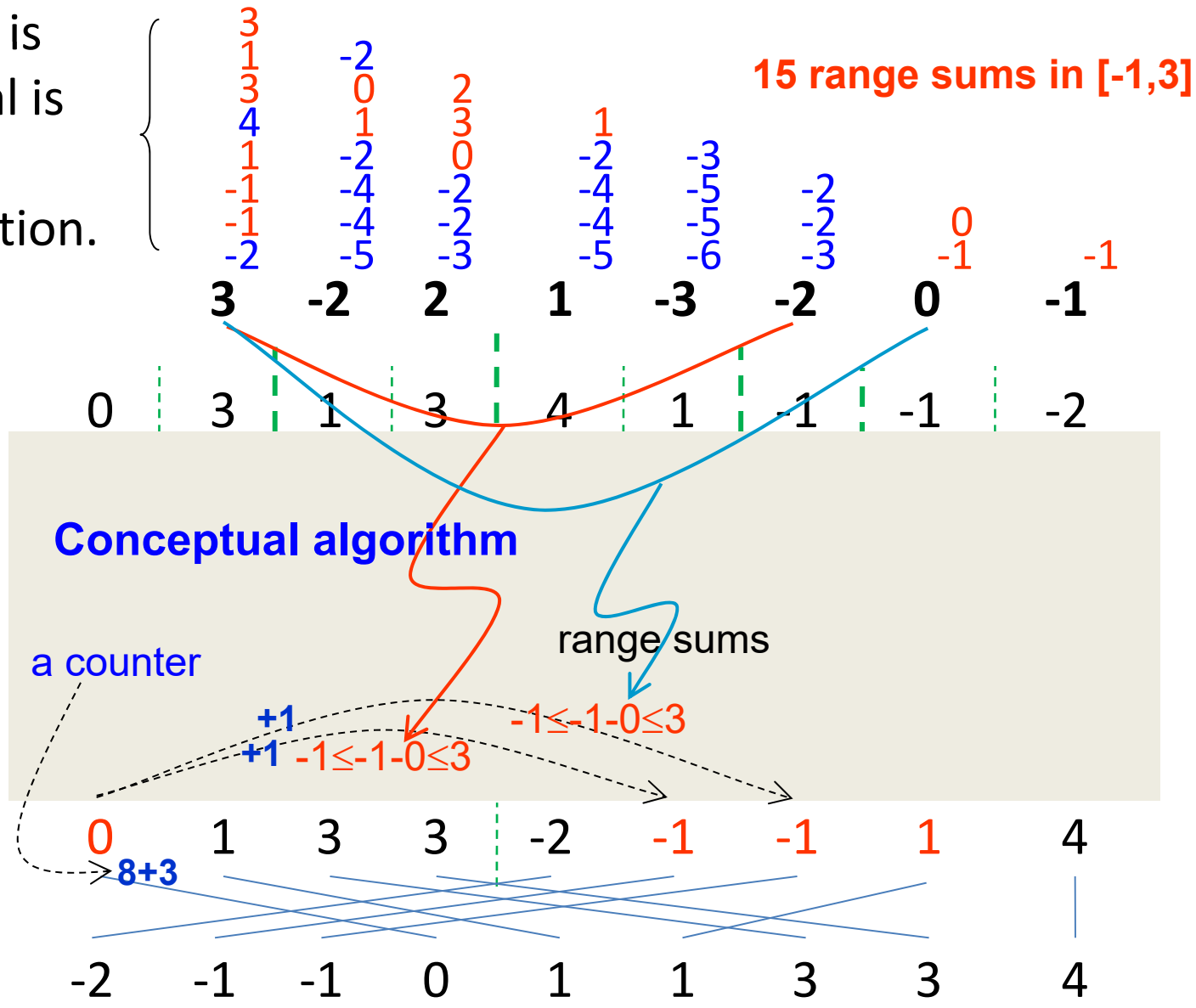

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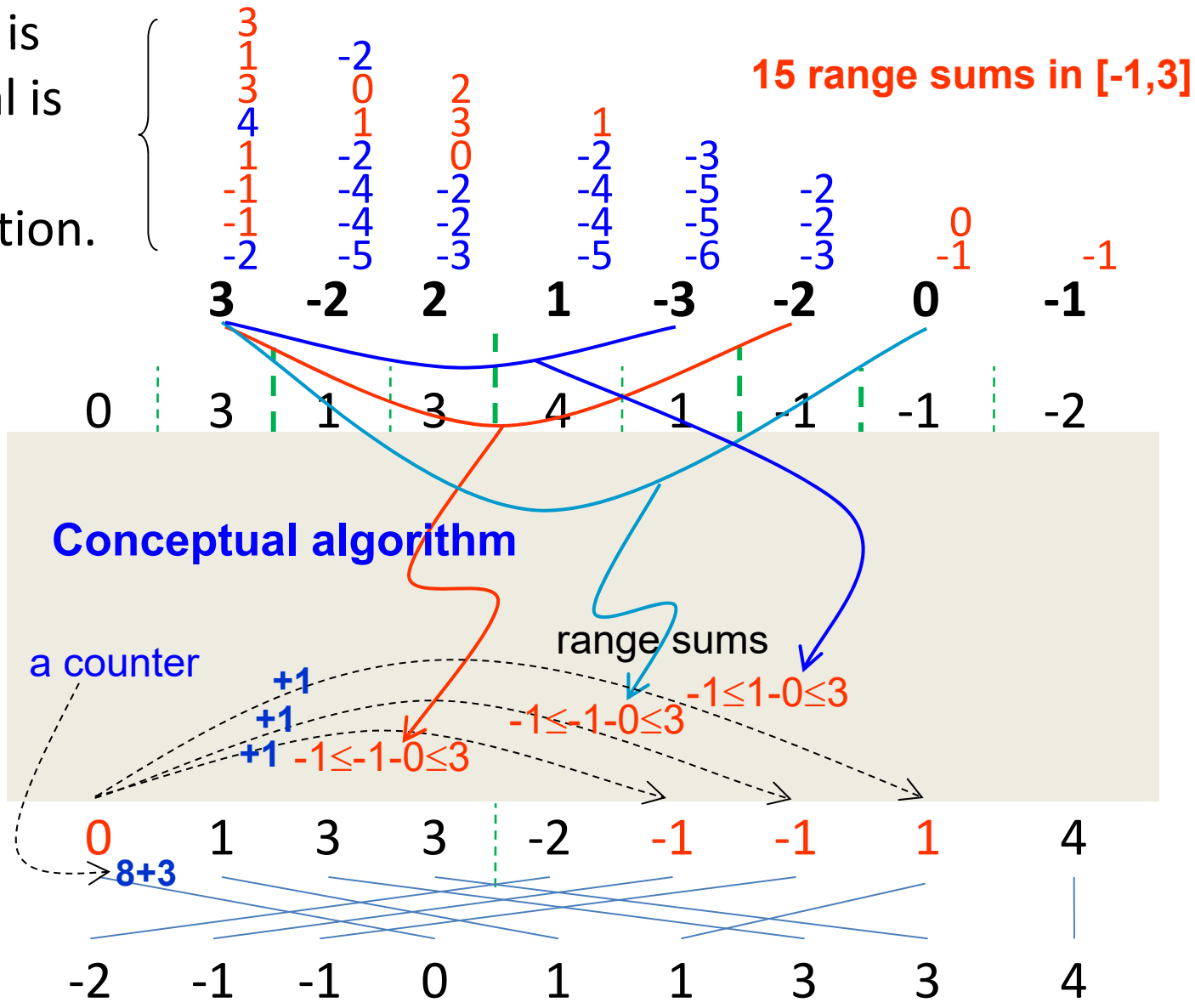

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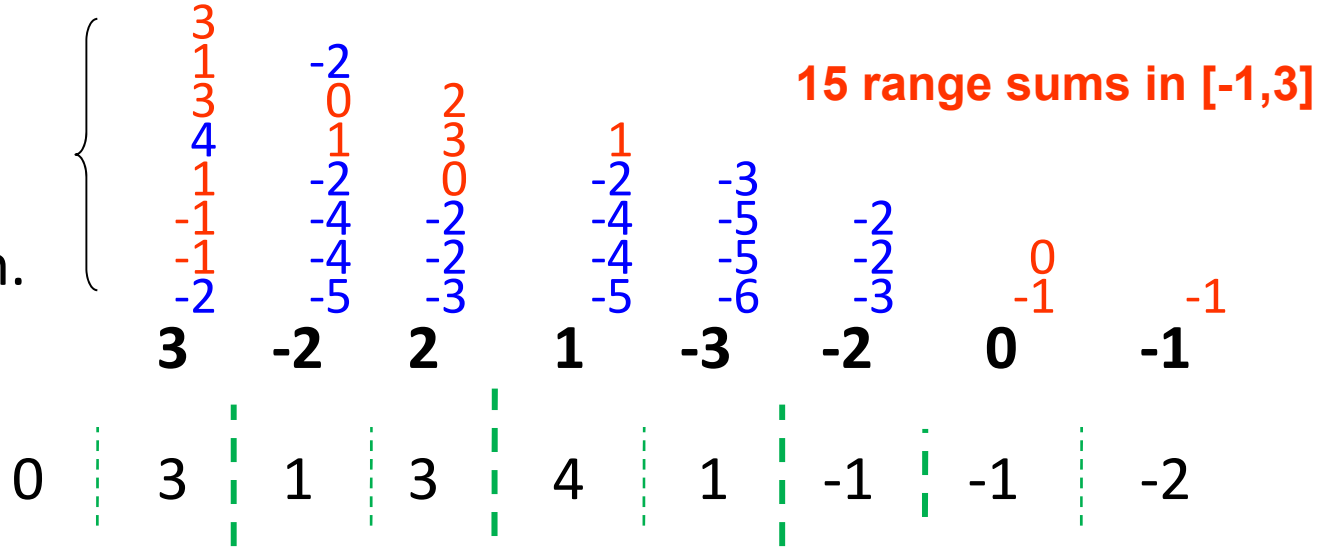

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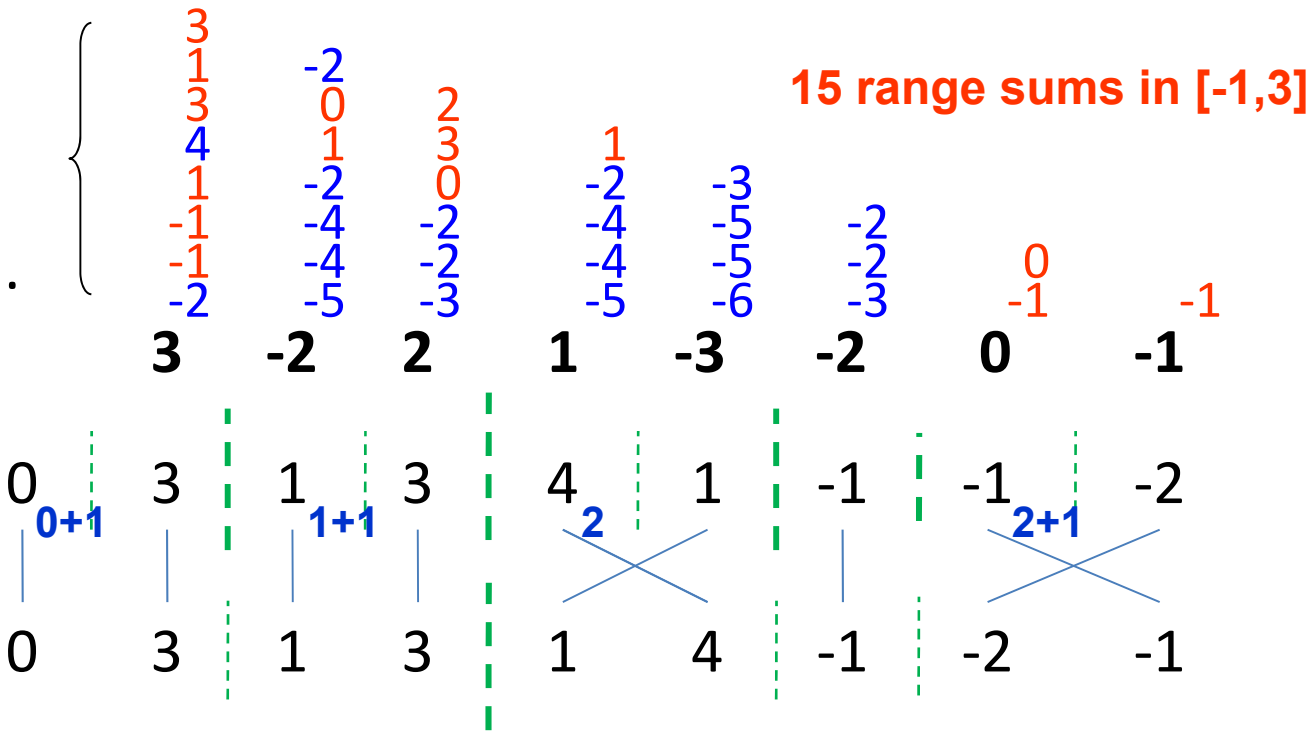

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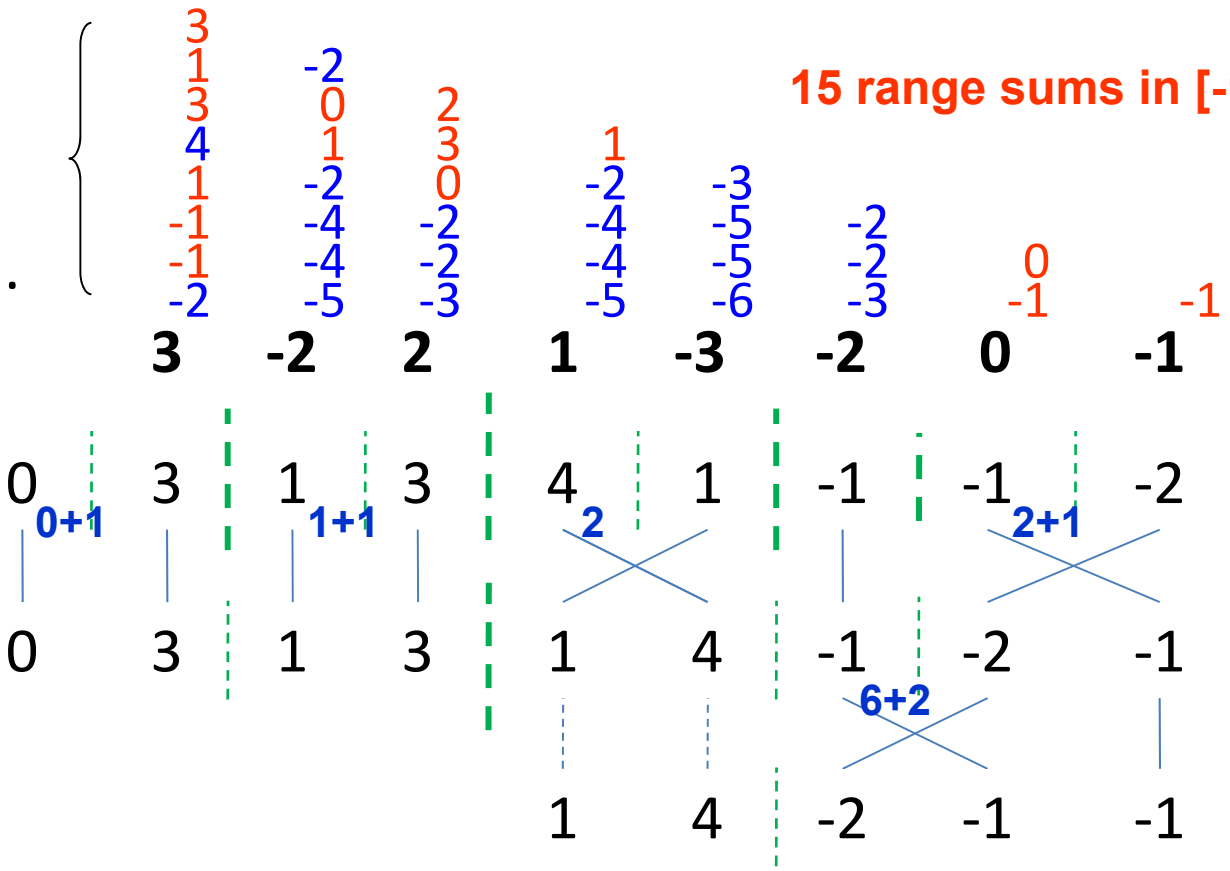

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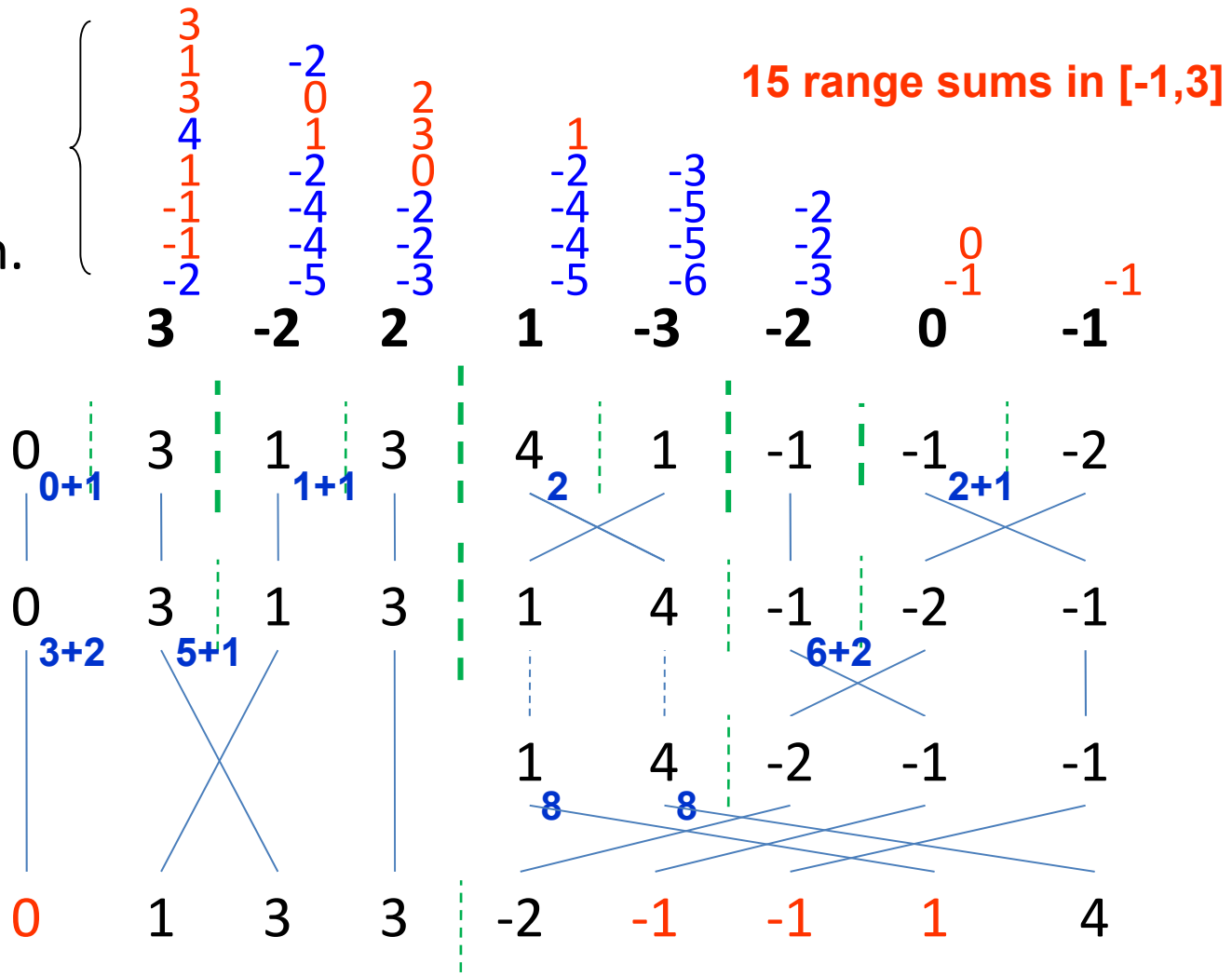

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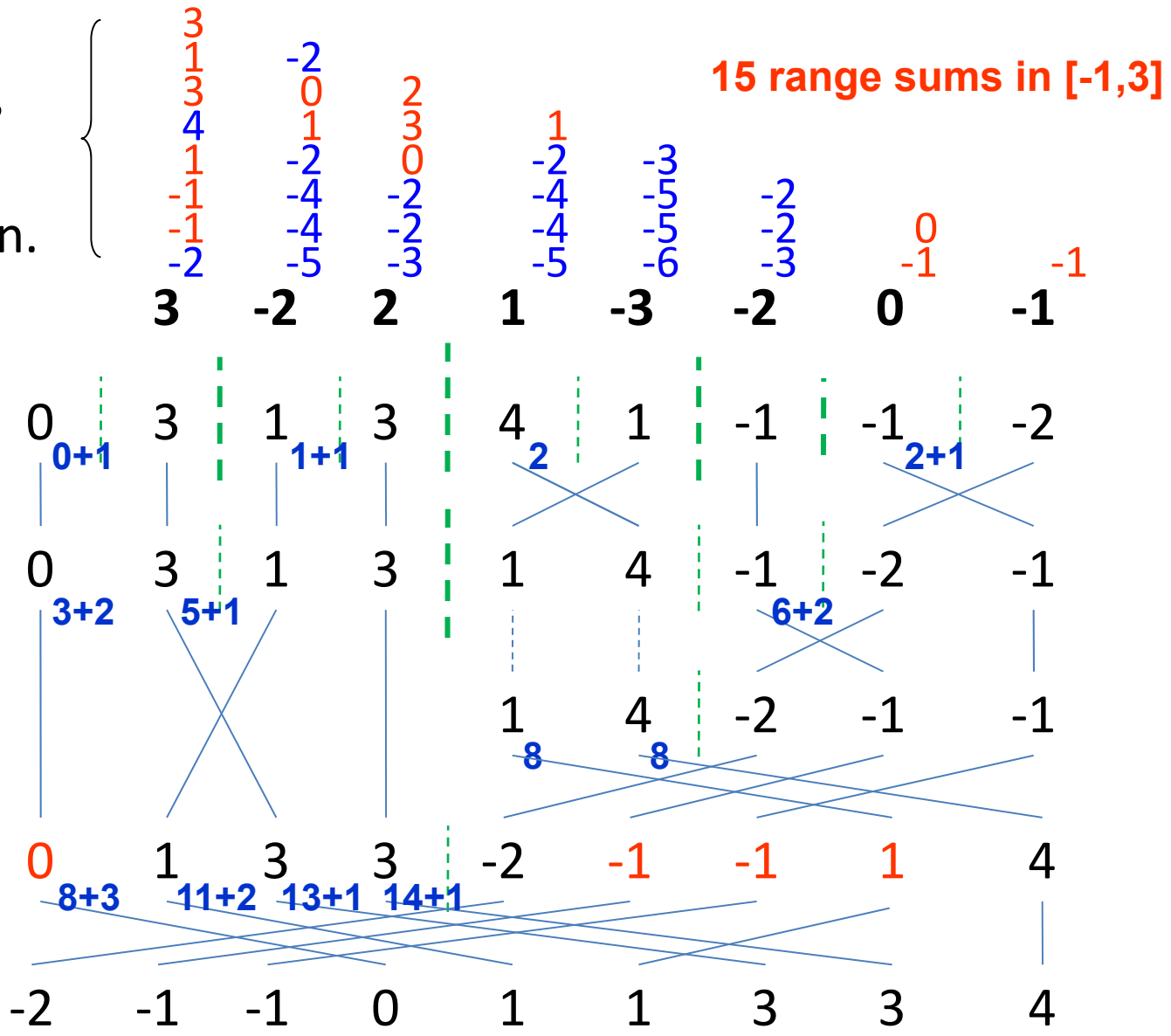

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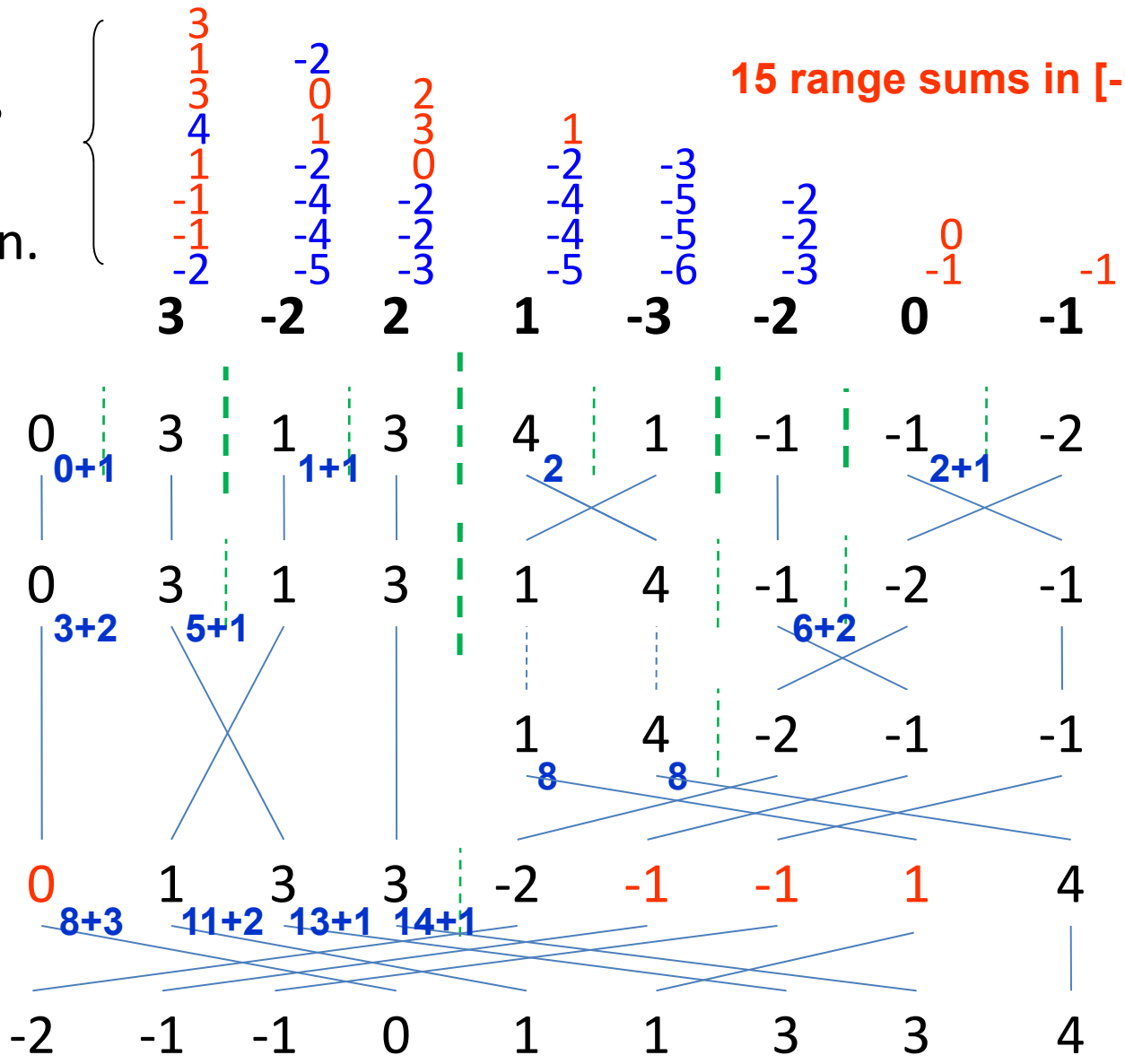

Why does it run in time O(n logn)?

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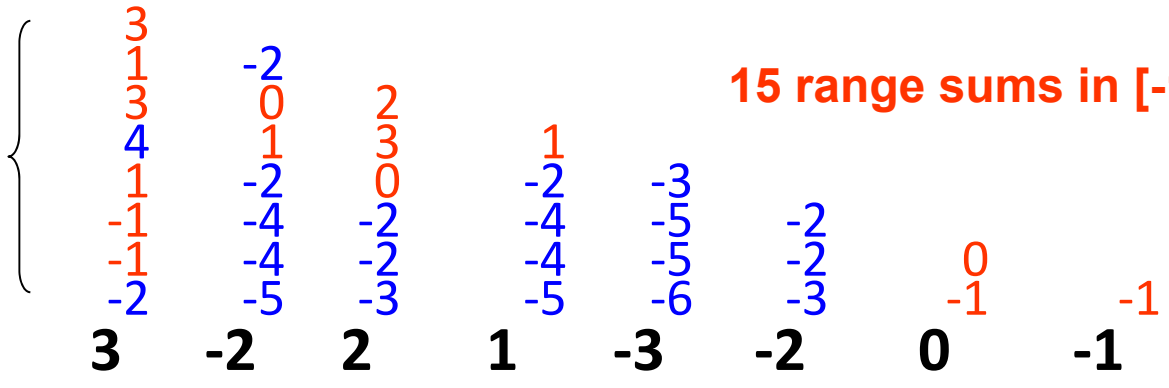
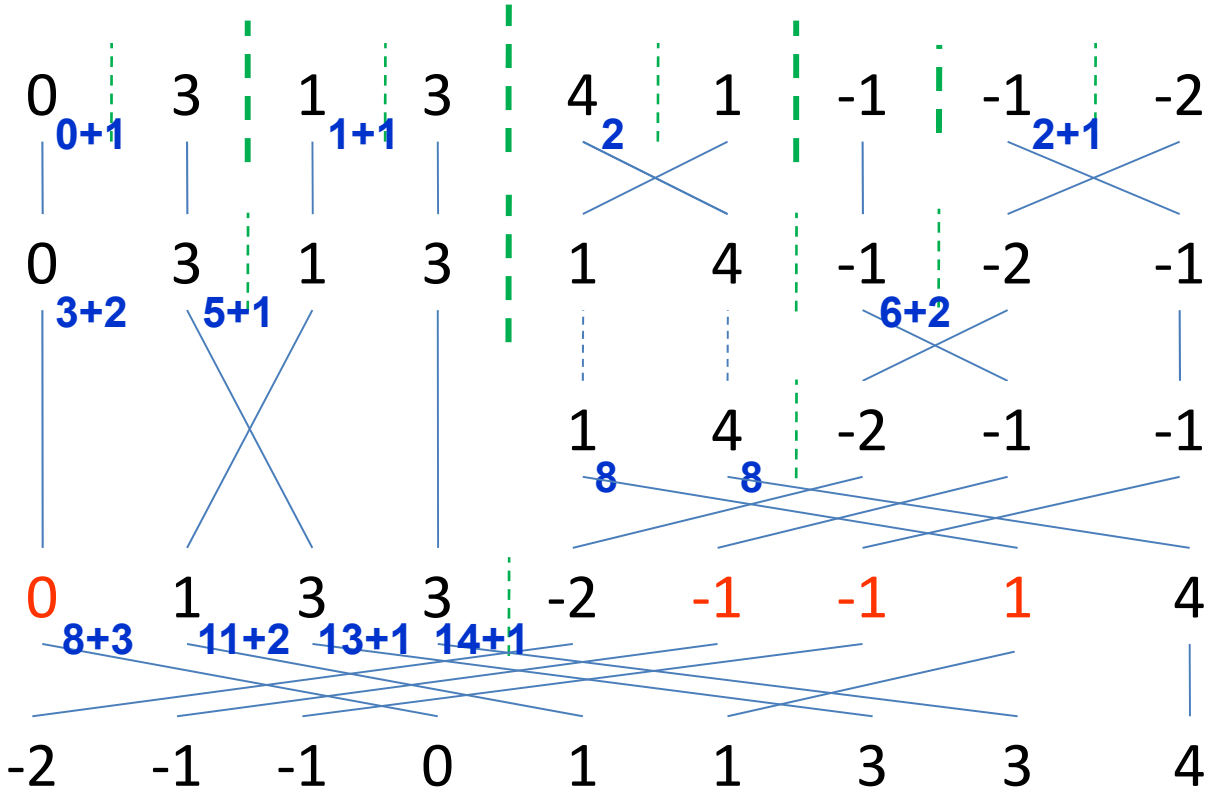


Why does it run in time O(n logn)?

Ans.

Some comparisons are skipped because they are ordered.

e.g. 0 ↔ -2
0 ↔ -1
0 ↔ -1
0 ↔ 1
0 ↔ 4
0<1 1 ↔ -2
1 ↔ -1
1 ↔ -1
1 ↔ 1
1 ↔ 4



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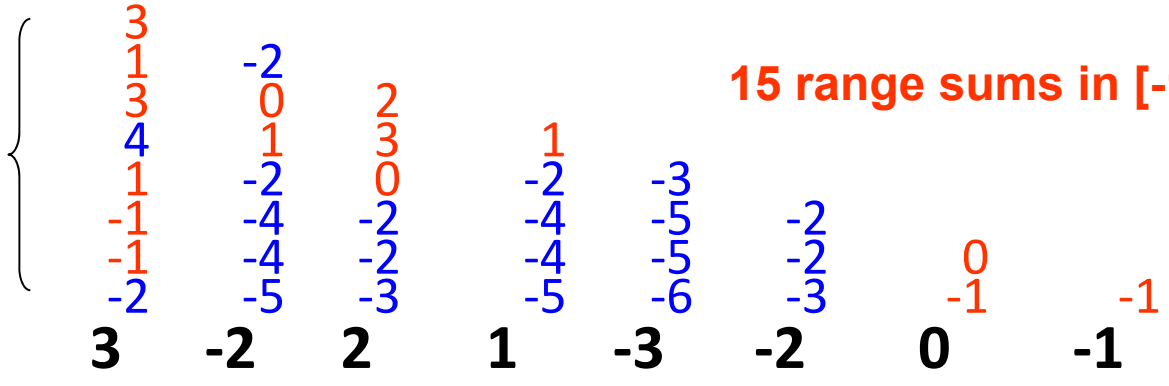

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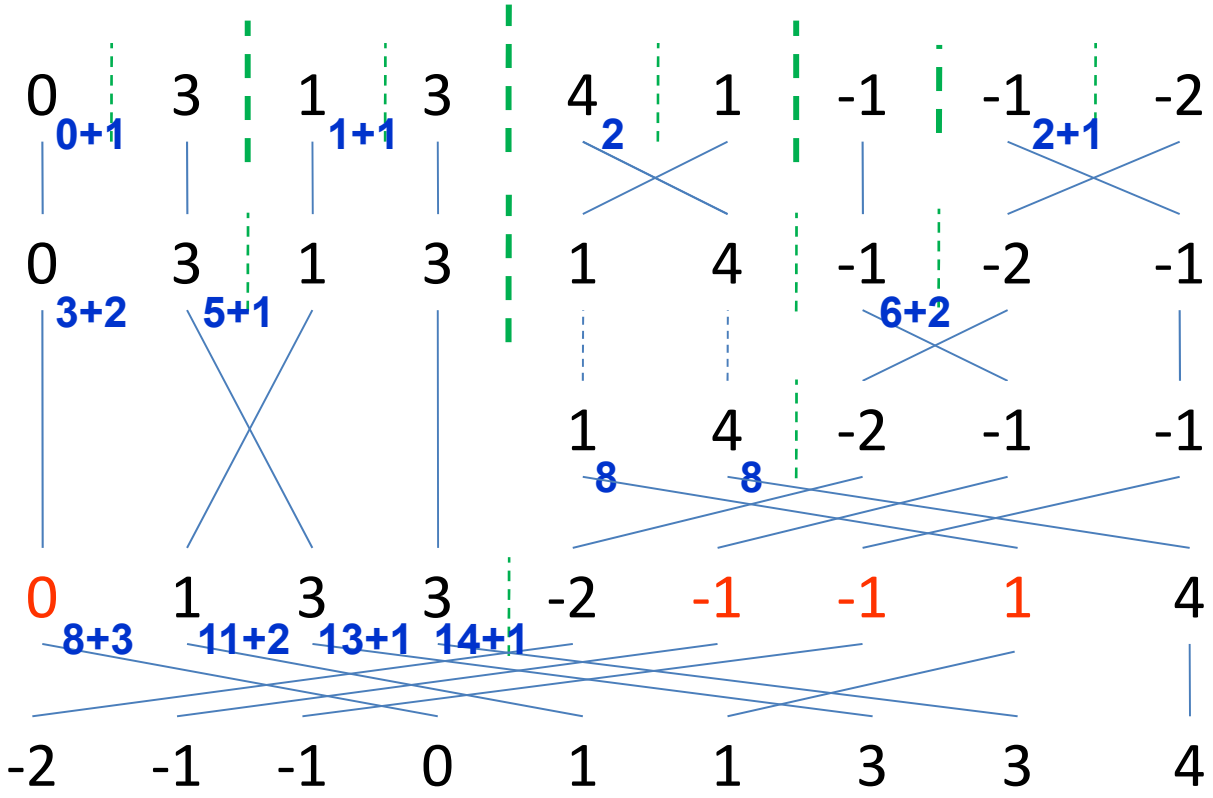
1 ↔ 1
1 ↔ 4



Not tied to the merge sorting algorithm.

LC327 Count of Range Sum

15 range sums in [-1,3]

3 -2 2 1 -3 -2 0 -1

min=-1
max=3

LC327 Count of Range Sum

15 range sums in [-1,3]

		3	**-2**	**2**	**1**	**-3**	**-2**	**0**	**-1**
min=-1									
max=3	0	3	1	3	4	1	-1	-1	-2

LC327 Count of Range Sum

15 range sums in [-1,3]

min=-1
max=3

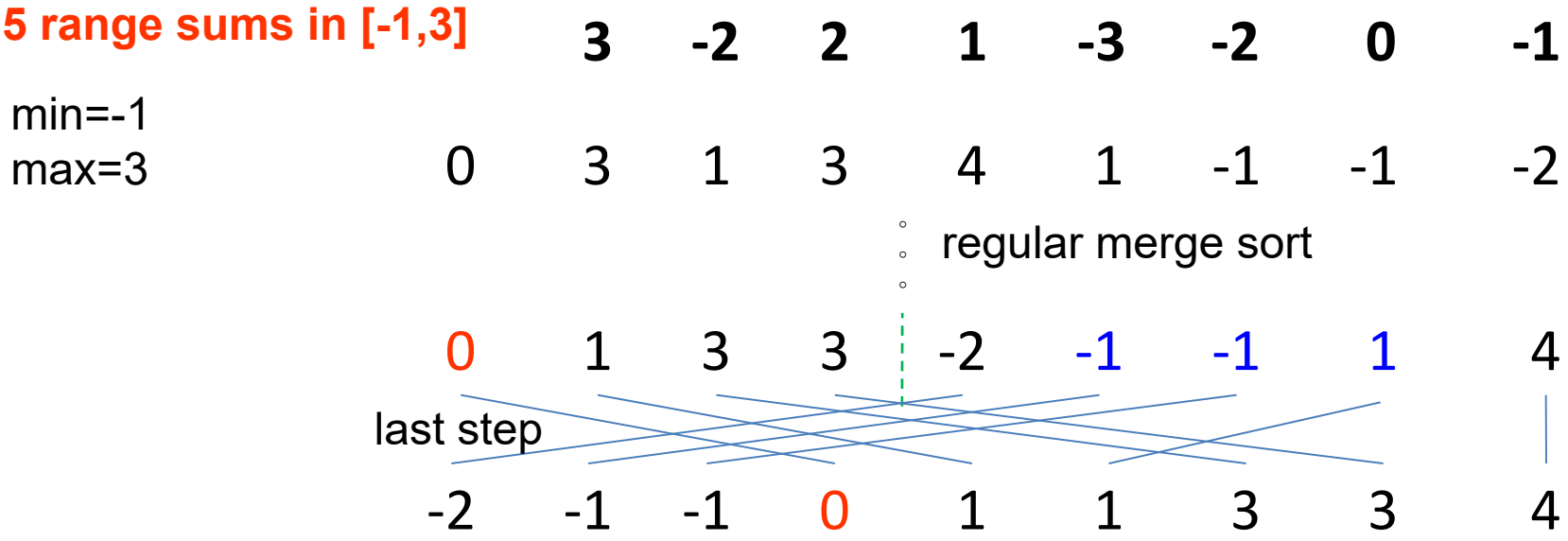
	3	**-2**	**2**	**1**	**-3**	**-2**	**0**	**-1**
0	3	1	3	4	1	-1	-1	-2

regular merge sort

0	1	3	3	-2	-1	-1	1	4

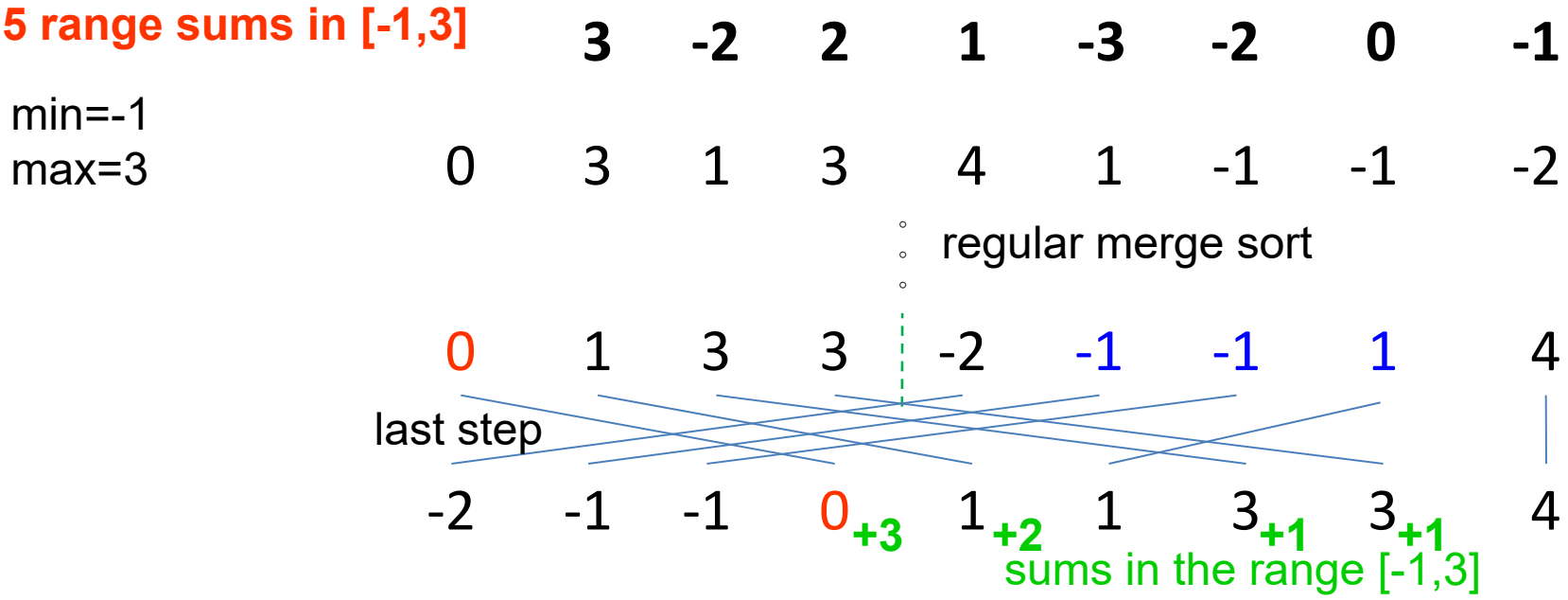
LC327 Count of Range Sum

15 range sums in [-1,3]

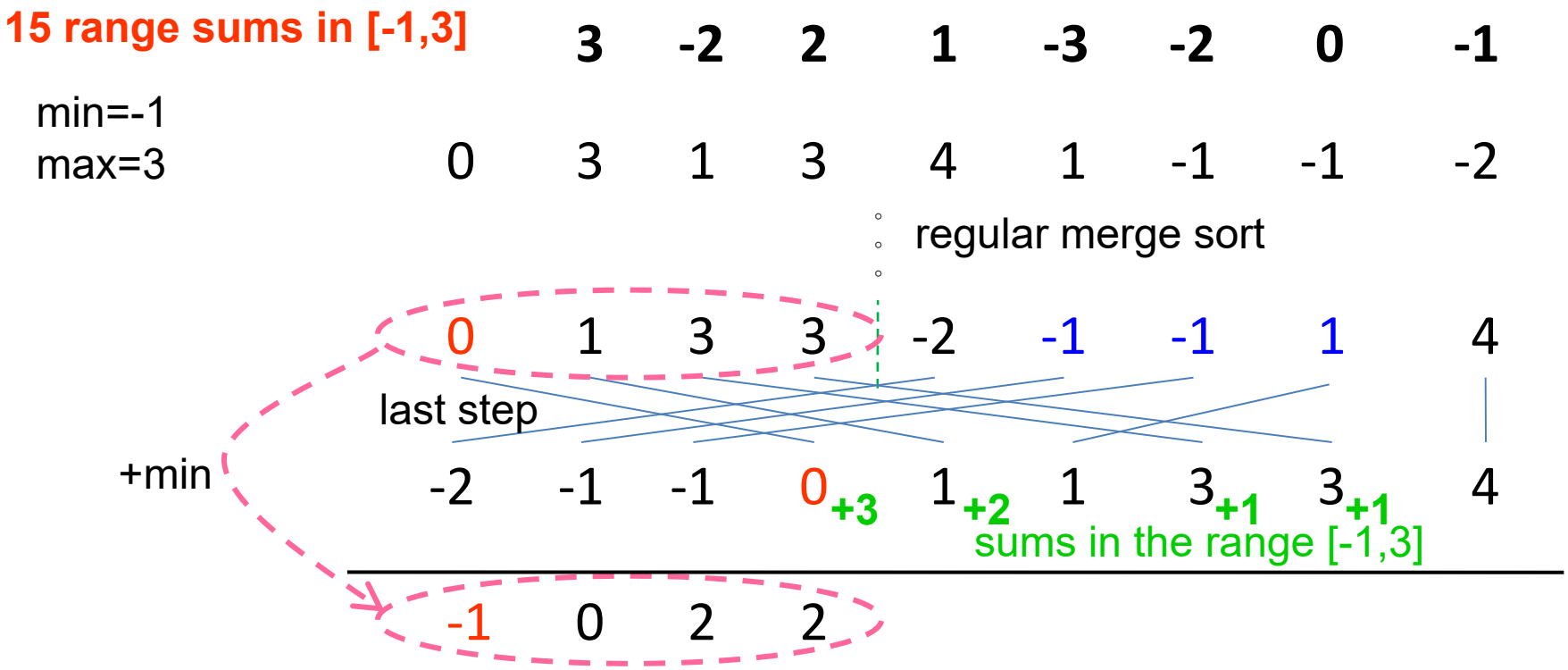


LC327 Count of Range Sum

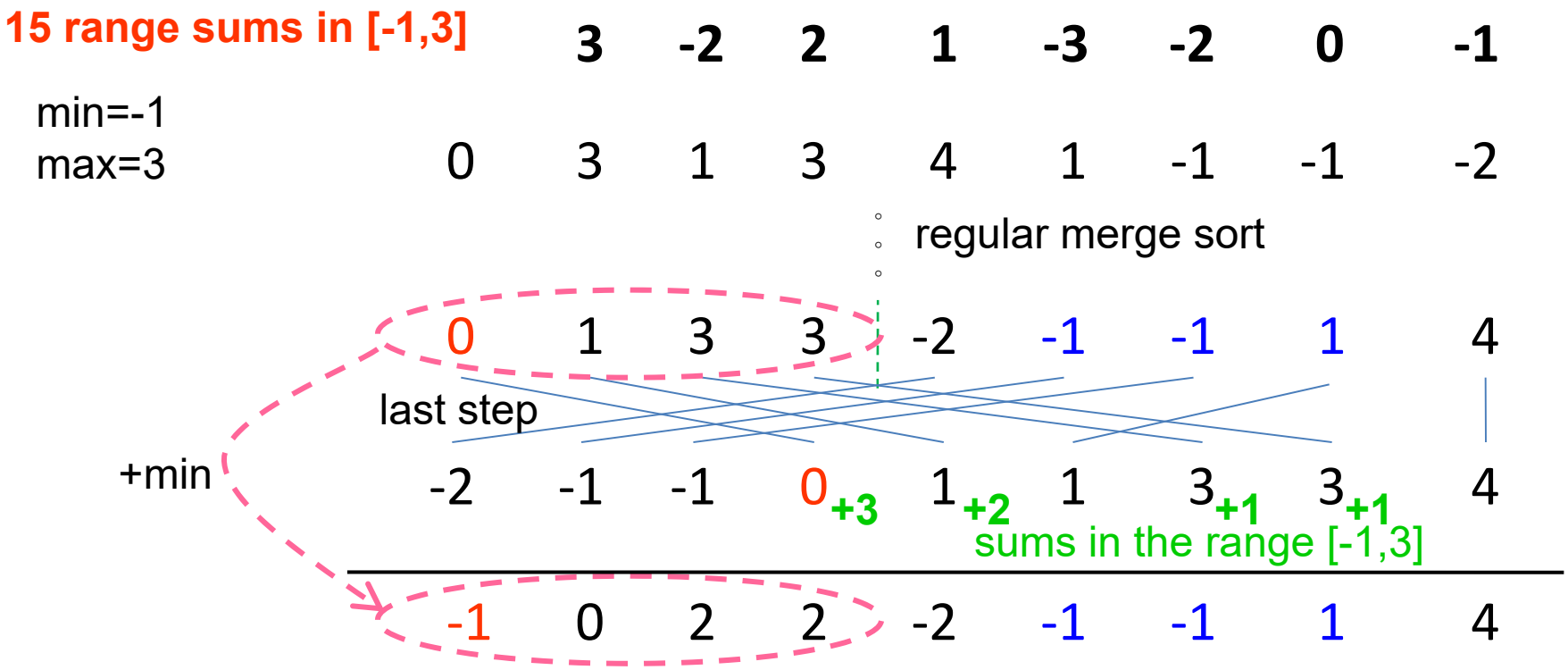
15 range sums in [-1,3]



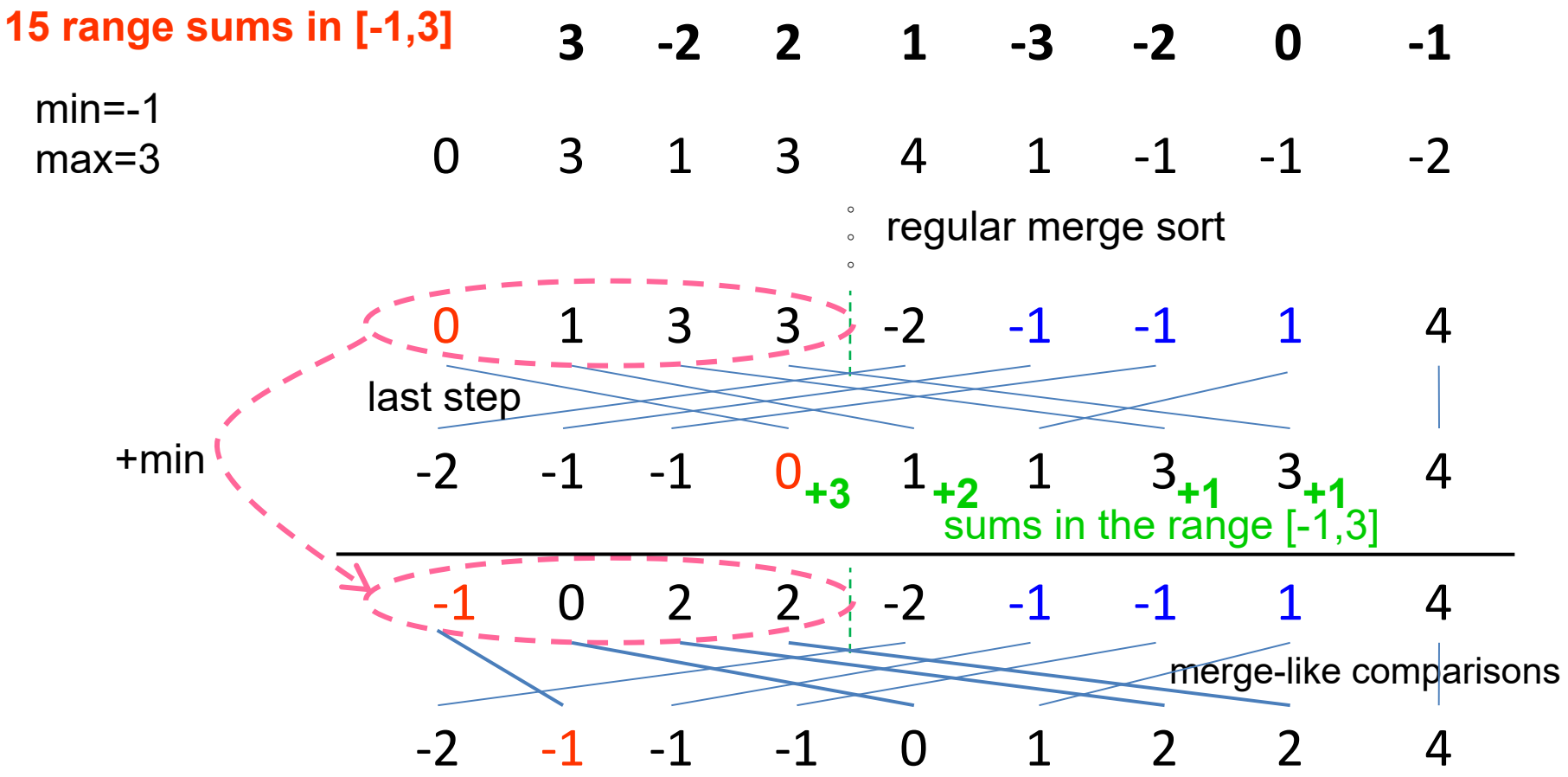
LC327 Count of Range Sum



LC327 Count of Range Sum



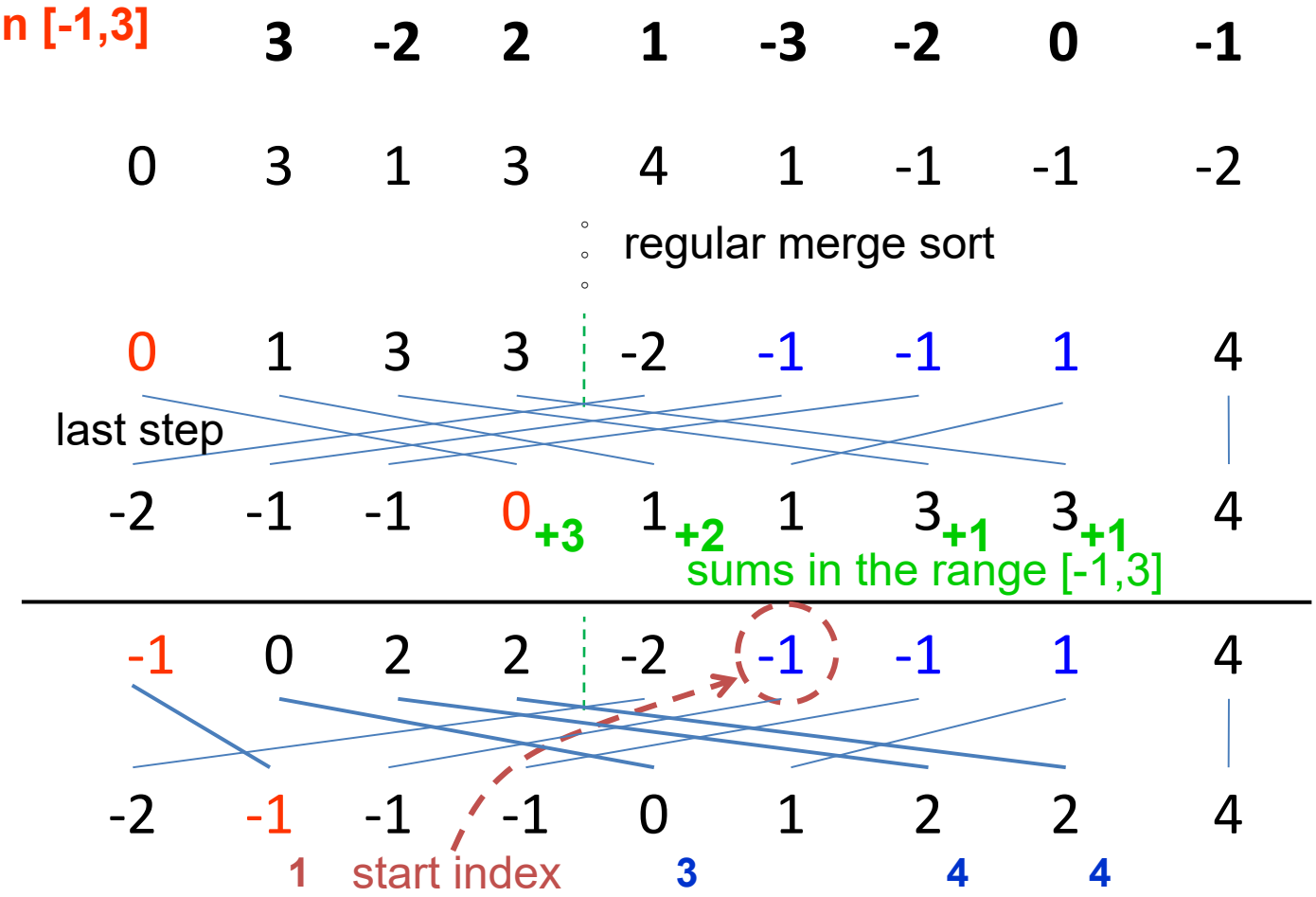
LC327 Count of Range Sum



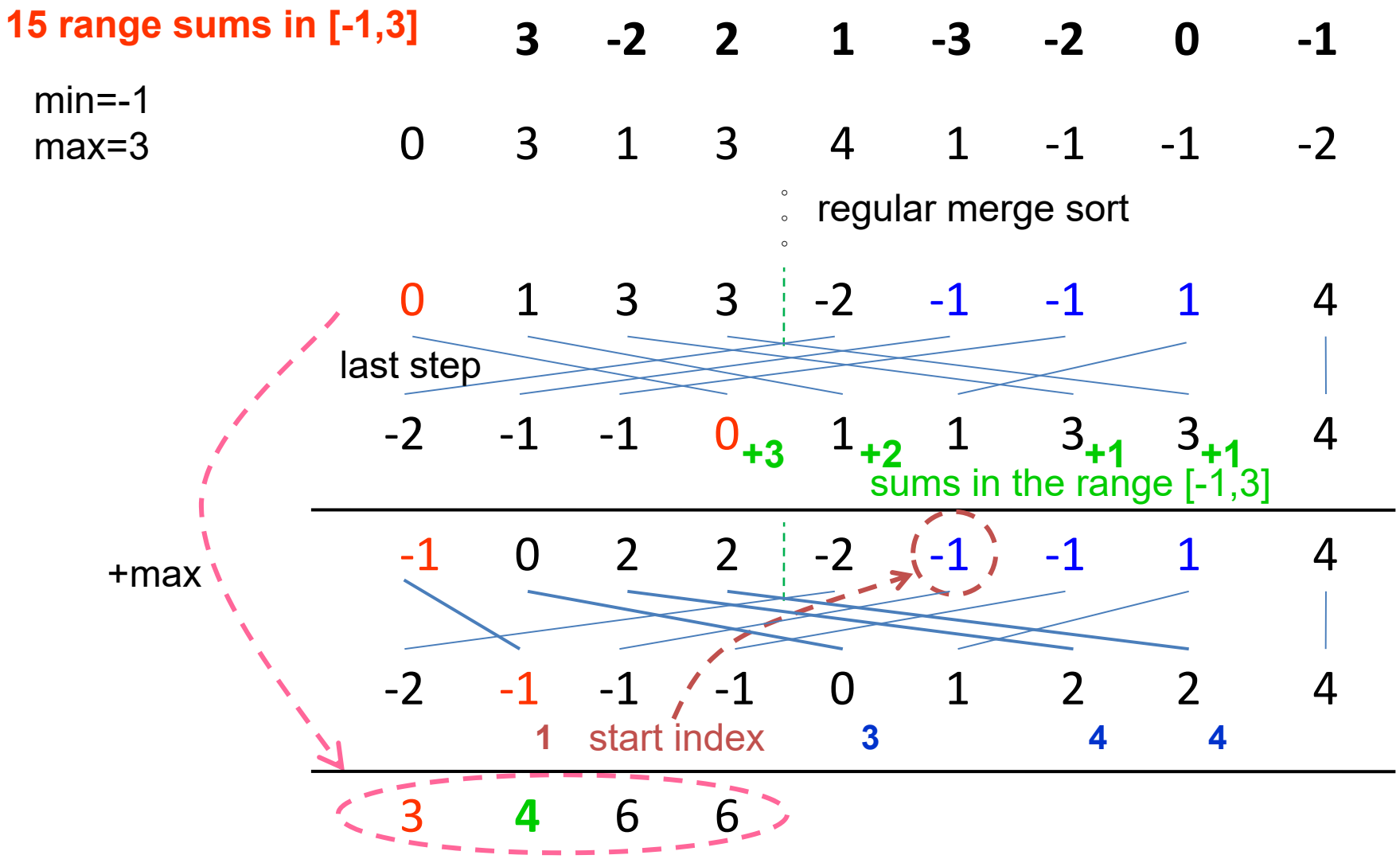
LC327 Count of Range Sum

15 range sums in [-1,3]

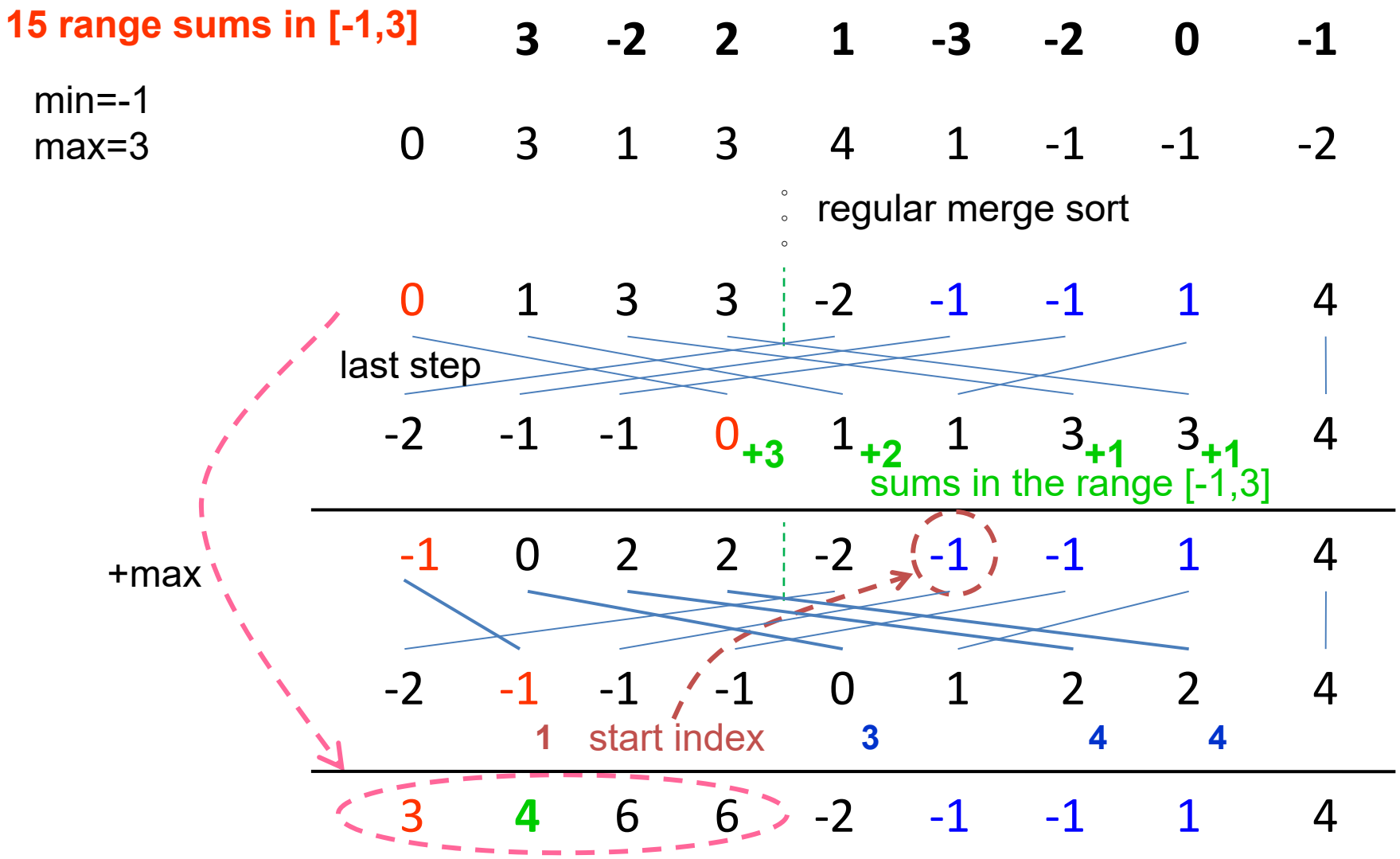
min=-1
max=3



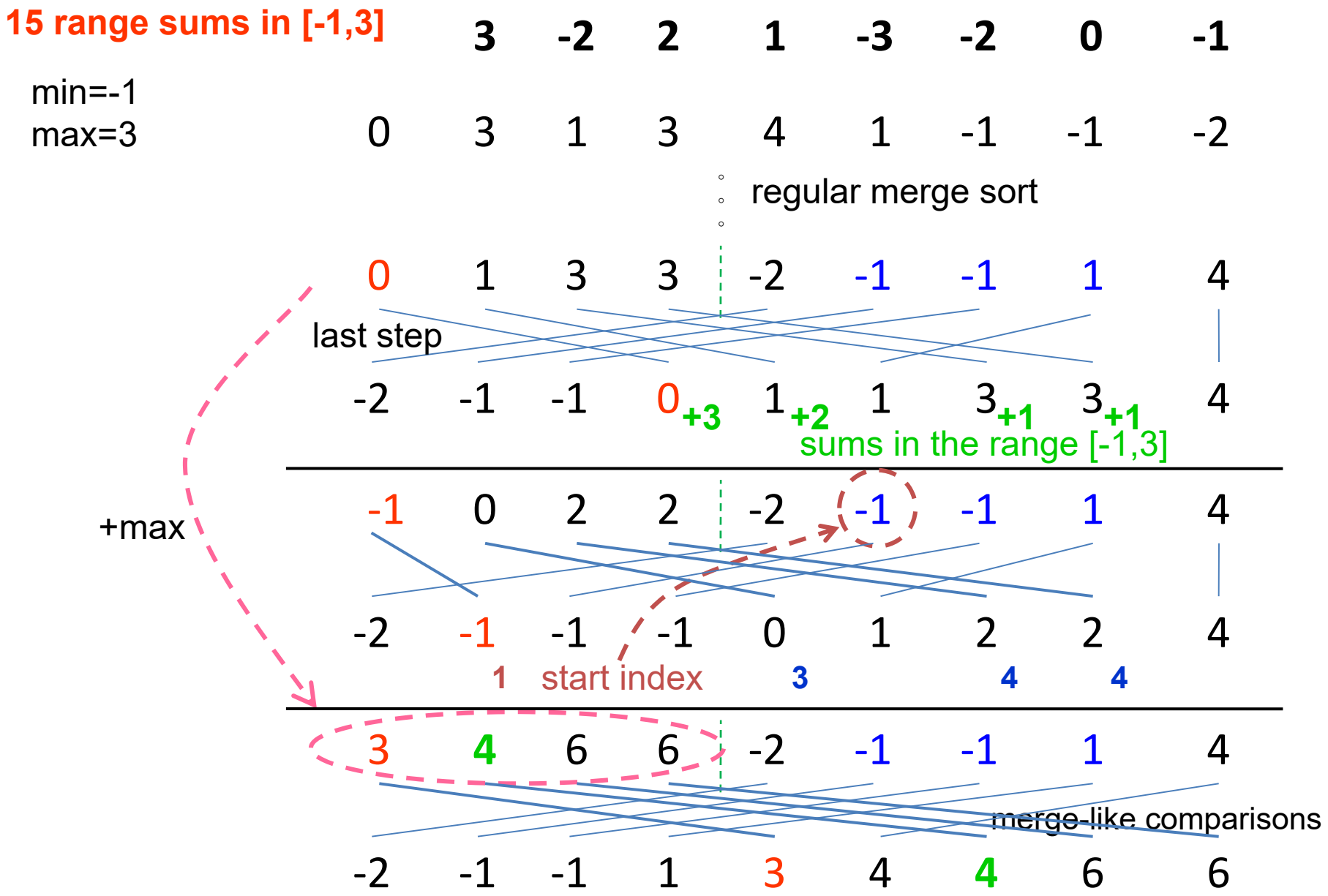
LC327 Count of Range Sum



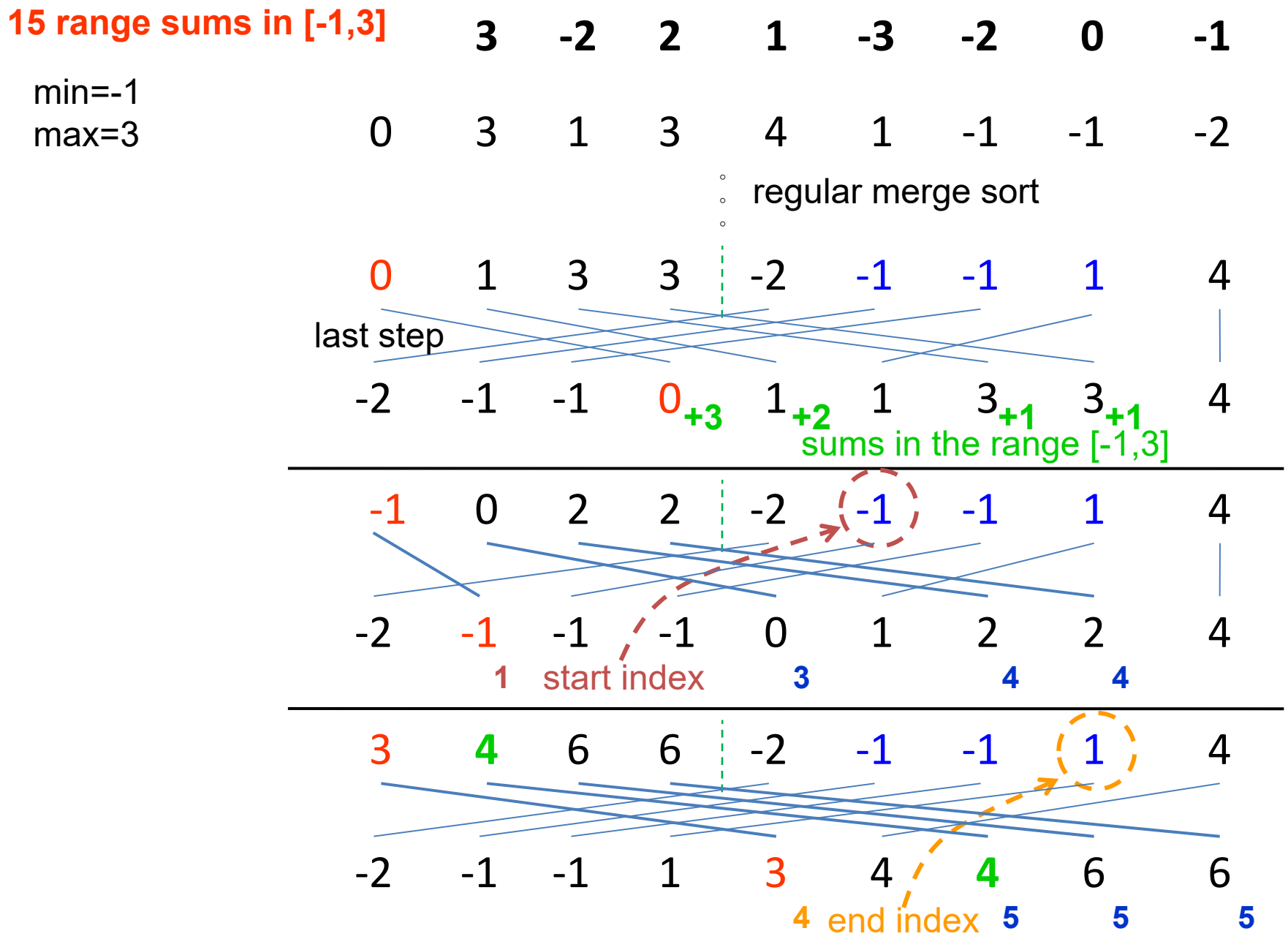
LC327 Count of Range Sum



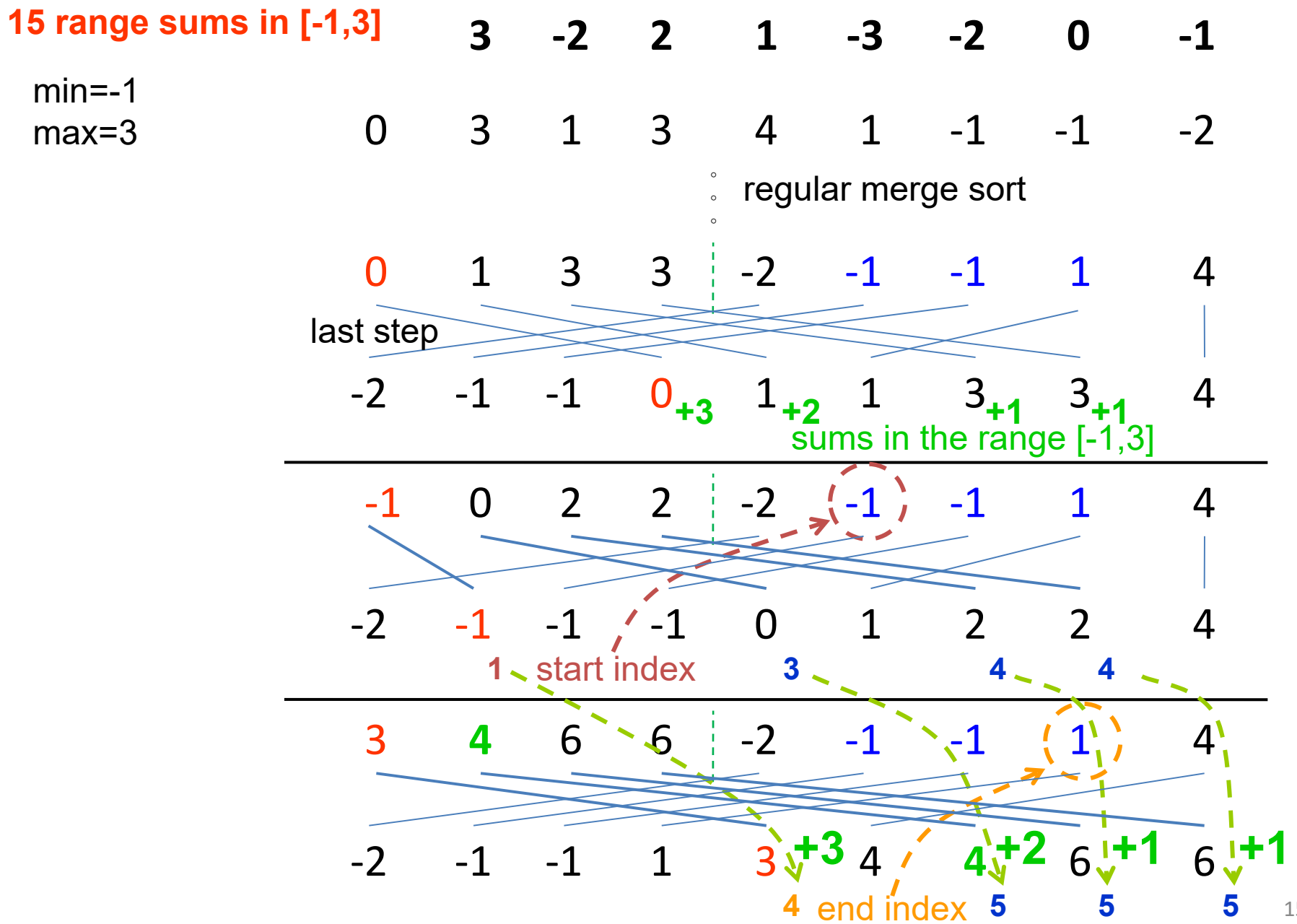
LC327 Count of Range Sum



LC327 Count of Range Sum

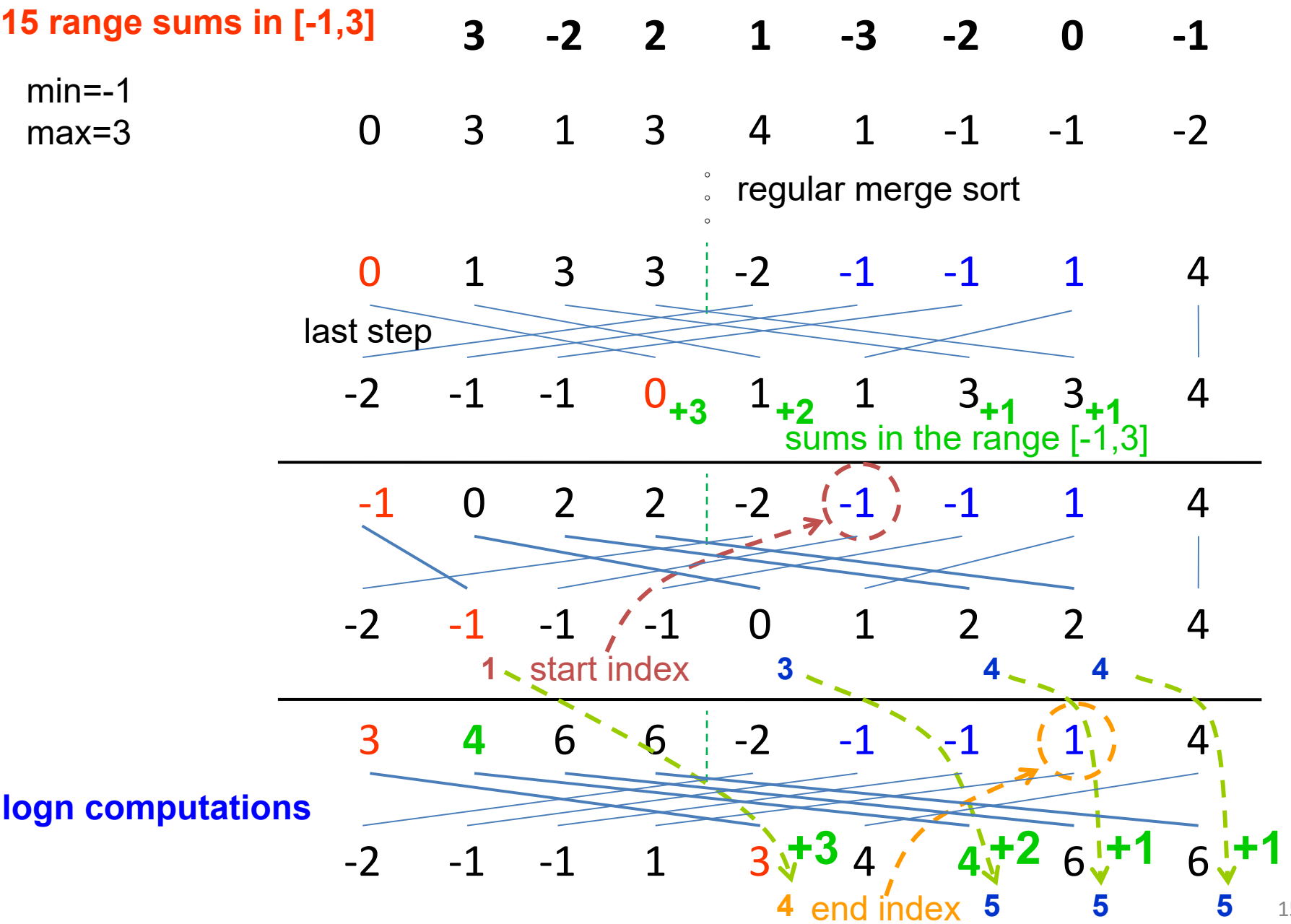


LC327 Count of Range Sum



LC327 Count of Range Sum

15 range sums in [-1,3]



3 n logn computations

其他應用

- ZJ d478. 共同的數 - 簡易版
- ZJ b512. 高維度稀疏向量